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FACT AND FICTIONS IN FX ARBITRAGE PROCESSES

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Fact and Fictions in FX Arbitrage Processes

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Abstract: The efficient markets hypothesis implies that arbitrage opportunities in markets such as those for foreign exchange (FX) would be, at most, short-lived. The present paper surveys the fragmented nature of FX markets, revealing that information in these markets is also likely to be fragmented. The "quant" workforce in the hedge fund featured in <u>The Fear Index</u> novel by Robert Harris would have little or no reason for their existence in an EMH world. The four currency combinatorial analysis of arbitrage sequences contained in Cross, Kozyakin, O'Callaghan, Pokrovskii and Pokrovskiy (2012) is then considered. Their results suggest that arbitrage processes, rather than being self-extinguishing, tend to be periodic in nature. This helps explain the fact that arbitrage dealing tends to be endemic in FX markets.

KEYWORDS: Arbitrage Sequences, Combinatorial Analysis, Asynchronous Systems, The Fear Index

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Introduction

An arbitrage operation involves taking advantage of discrepancies in the prices at which a particular good or asset can be bought or sold, such operations being distinct from speculation in that they involve minimal risk or capital exposure. In the economics and finance literature the "law of one price" postulates that the exploitation of opportunities to profit from arbitrage, if buying prices are lower than selling prices, will lead goods or assets that are in some sense "identical" to have the same price.

The present paper focuses on arbitrage operations in foreign exchange (FX) markets. The Bank for International Settlements (BIS 2011, p. 5) offers a useful distinction between different arbitrage strategies in FX markets. *Classical arbitrage* exploits differences between quoted exchange rates and the exchange rates implied by no-arbitrage conditions, such as that cross-exchange rates are consistent with the exchange rates for currency pairs. *Latency arbitrage* takes advantage of time lags between market-moving trades being initiated and market-makers updating their FX price quotes. *Liquidity imbalance strategies* aim to detect order book imbalances for currency pairs and pricing discrepancies between different trading platforms. *Complex event* processing aims to exploit properties of foreign exchange rates such as momentum, mean-reversion, correlation with other exchange rates or reaction to news releases. The present paper focuses on *classical arbitrage strategies*.

Section 1 of this paper considers the institutional context in which FX arbitrage strategies can be executed. In contrast to the textbook fiction of a single FX market, there are a variety of FX trading platforms distinguished not only by geographical location and time zone but also by the means by which transactions are executed. Electronic FX trading platforms have grown in importance in the last decade, but a substantial proportion of transactions are still conducted by banks dealing directly with end-user customers. This leads to a contrast between the "lit" areas, where information on FX quotes and trades is publically available on some of the electronic trading platforms; and the "dark pools", where information on customer-direct transactions is initially private. A further contrast is between the

human and computer elements. At one extreme are arbitrage strategies formalised in computer algorithms and executed by computer algorithms. At the other extreme are trades where humans are in the strategy and execution driving seats. So the notion of a single FX market is more fiction than fact.

Section 2 considers the account given of FX markets by the efficient markets hypothesis (EHM) that came to be the conventional academic wisdom as to how financial markets work. The EMH postulates that prices, such as those in markets for FX, will fluctuate randomly if they incorporate the information and expectations of all market participants (Samuelson 1965). Weak, semi-strong and strong variants of the EMH are distinguished (Fama 1970) depending on whether the information available involves just the past history of market prices; other publically available information as well; or privately available information as well. If the EMH held in its strong form, opportunities to profit from arbitrage would be fleeting and would not persist in the absence of continuing information surprises (Fisher 1989). We argue that the EHM belies too many facts for comfort.

Section 3 considers the account of financial markets in the novel *The Fear Index* (Harris 2011). In this thriller a particle physicist leaves his research post at the Large Electron-Positron Collider at CERN, Geneva to become a "quant", partnering a banker to set up a hedge fund based on an algorithmic-learning trading strategy. The fictional VIXAL-4 fund makes huge profits by *complex event processing* arbitrage based on algorithms "learning" how fear indices affect asset prices. Aspects of the psychology and neuroscience literature dealing with financial markets are considered in an attempt to identify a factual basis for this novel.

Section 4 considers *classical arbitrage* sequences in FX markets. The starting point is a three-currency world in which one arbitrage transaction suffices to exploit and eliminate FX price discrepancies. The results for a four-currency world are then discussed, the periodicity implications of the combinatorial analysis of arbitrage sequences in asynchronous systems in Cross, Kozyakin, O'Callaghan, Pokrovskii and Pokrovskiy (2012) being highlighted. The implications of moving to a five or higher order currency world Cross and Kozyakin (2012) are then considered. The *Financial Times* provides daily quotes for fifty-two currencies, so the higher order currency worlds are worthy of consideration.

Section 5 offers some concluding remarks.

1. FX Markets

The Bank for International Settlements conducts a triennial survey that provides useful documentation on global FX market activity (BIS 2010). The UK is the dominant location for FX market activity, accounting for 37% of global turnover, followed by the US (18%), EMU countries (10%), Japan (6%), Singapore (5%), Switzerland (5%), Hong Kong (5%) and Australia (4%). There tends to be a "home bias" between the different time zones, most trades initiated in the US and Canada occurring during North American trading hours, most trades initiated in Japan and Australia taking place during Asian trading hours, and UK-initiated trades tending to be bunched in the overlapping Europe-North America and Asia-Europe trading hours (D'Souza 2008, Table 2).

FX trades involving the (US) dollar account for 85/200 of all trades, followed by the euro (39/200), yen (19/200) and sterling (13/200), so trades involving these four currencies account for 78% of global turnover (BIS 2010, p. 12). In terms of currency pairs the dollar-euro accounts for 28% of global turnover, followed by the dollar-yen (14%), dollar-sterling (9%), euro-yen (3%), euro-sterling (3%) and sterling-yen (0.5%), so these six pairs account for 57.5% of global turnover (BIS 2010, p. 15). Triangular arbitrage operations involving these currency pairs are considered in the four-currency case analysed in Section 4 of this paper.

Over the last decade there has been a substantial increase in the proportion of FX transactions that are executed electronically (BIS 2010, p. 16). Electronic broking systems, such as the London-based EBS and Reuters, accounted for 18.8% of turnover. Multi-bank electronic communication networks (ECNs) such as the US-based Currenex, Hotspot FX and FXall, accounted for 11.1% of turnover, with single-bank ECNs accounting for 11.4% of global turnover. Most inter-dealer trades are also executed electronically, accounting for 18.5% of turnover. The non-electronic execution part of the market is made up of customer-direct transactions, where banks deal directly with end-user clients, accounting for 24.3% of turnover; and voice broker transactions, which hark back to the method of

executing transactions by telephone, which was the predominant method of executing FX transactions until the late 1980s, taking 15.9% of turnover (BIS 2010, p. 16).

In the 1990s electronic trading was confined to the inter-dealer section of the FX market. The growth of electronic trading in the 2000s has seen the emergence of new trading platforms and new participants in FX markets. The growth of high-frequency trading (HFT) has been one of the most notable developments. Algorithms can be used to execute trades, such as by "splitting trades to minimise the footprint on the market" (BIS 2011, p. 3); or to make trading strategy decisions with the algorithm being a *classical arbitrage, latency arbitrage, liquidity imbalance or complex event processing model*.

HFT firms use algorithms for decision-taking as well as execution of FX trades. The latency, that is the delay between the transmission of information from a source and the reception at its destination, of HFT trades can be less than a millisecond, 10^{-3} seconds – compared to around 150 milliseconds for a human blink (BIS 2011, p. 4). HFT firms trading FX tend to operate on high volumes and low margins. Access to infrastructure support is by prime brokerage arrangements with large investment banks, as is credit, though risk holding periods are "typically well under five seconds" (BIS 2011, p. 1). On March 2011 the EBS introduced a fifth decimal to the pricing of the major currency pairs, this finer grain "pip" being easier to handle for the algorithms than for the human traders. The estimate is that automated HFT accounted for 24-30% of total spot FX turnover in 2010 (BIS 2011, p. 11).

A pioneering study by Chaboud, Chiquoine, Hjalmarsson and Vega (2009) used one-second frequency data from the EBS platform to assess how computer and human-driven FX trades interact, distinguishing between computer-maker/computer taker, computer-maker/human-taker, human-maker/computer-taker and human-maker/human-taker trades. They found that computer-driven trades tend to be more correlated than human-driven trades, suggesting that the algorithms employed are not very diverse. There was no evident relationship between computer-driven trades and higher exchange-rate volatility, and non-algorithmic order flow accounted for a higher share of the variance in FX trading returns than algorithmic order flow.

2. The EMH and FX Markets

If the EMH holds, foreign exchange rates would fluctuate randomly provided that they fully incorporate the information and expectations of all market participants (Samuelson 1965). To test the EMH, auxiliary hypotheses need to be invoked to specify, inter alia, the information sets available. Fama (1970) distinguished three variants of the EMH that could be subjected to statistical tests. In the weak version only information about present and past values of, in this context, exchange rates is involved: so technical analysts or algorithms would not be able to make arbitrage profits by attempting to identify patterns in past exchange rate movements, or discrepancies between exchange rates for currency pairs and their cross-exchange rates. In the semi-strong form the EMH involves publicly available information about the "fundamentals" that determine exchange rates as well: so technical analysts and algorithms would not be able to profit from using macroeconomic news, order flow information available on electronic trading platforms or other relevant driving forces when conducting arbitrage operations. In the strong variant of the EMH, privately available information is also reflected in actual exchange rates: so, for example, FX traders at an investment bank would not be able to make arbitrage profits by exploiting the order-flow information arising from FX trades conducted directly with end-user clients.

Thus, under the weak version of the EMH, exchange rate movements should be random, and arbitrage possibilities arising from discrepancies between the exchange rates for currency pairs and their cross exchange rates would not exist. Under the *semi-strong* and *strong variants* of EMH, technical analysts or computer algorithms would not be able to detect systematic relationships between exchange rates and, respectively, public or public and private information sets. This raises the "arbitrage paradox" pointed out by Grossman and Stiglitz (1976). If arbitrage opportunities do not exist, FX market participants would not have any incentive to monitor exchange rate movements, in which case profitable arbitrage opportunities could well arise. The informal resolution of this paradox by adherents of the EMH is to suggest that arbitrage opportunities arise over short time intervals and are quickly eliminated. Given that the HFT firms can execute trades within a 10^{-3} second interval on electronic FX trading platforms, the short-time interval would have to be very short indeed. A further basic prob-

lem with the EMH is the "no trade" theorem (Grossman and Stiglitz 1980). If markets such as those for FX were informationally efficient, one would expect to observe periods in which transactions volumes vanished towards zero when there is no new information coming on stream. Instead FX transactions, although varying over time, remain substantial during quiescent market conditions.

An influential paper by Meese and Rogoff (1983) produced evidence, for low-frequency exchange rate data, that models in which macroeconomic "fundamentals" drive exchange rates did not forecast any better than the EMH postulate that the changes are random. Engel and West (2005) showed that exchange rates would display something close to a random walk if the "fundamentals" followed an I(1) process and the factor for discounting future fundamentals was close to one.

The microstructure literature on the FX market has focused attention on the role of order-flow information in explaining exchange rate changes. Some of this information is private, arising, for example, from FX transactions in which banks deal directly with end-user clients. Evans and Lyons (2005) showed that end-user order flow data can explain about 16% of the monthly spot rate variance between the dollar and the euro. Covrig and Melvin (2002) studied whether geographically local order flow conveys informational advantages. Until December 22, 1994 the Japanese FX market closed for lunch from 12.00 to 13.30 hours, Tokyo time. The authors found bunching of FX transactions before this lunchtime closure, and a significant tendency for foreign quotes for the yen-dollar exchange rate to lag behind Tokyo quotes during this pre-lunch period. This suggests that Tokyo based traders were better informed, or that foreign-based traders believed this to be the case. Osler (2005) discovered significant effects of stop-loss order flows on minute-by-minute foreign exchange rate data, a stop-loss order being an instruction to buy(sell) at the FX platform's spot rate once this has risen (fallen) to a pre-specified level.

Studies of high frequency data on electronic trading platforms suggest that arbitrage trading profits could have been made, net of transactions costs. Data from the EBS platform were studied by Marshall, Treepongkaruna and Young (2007) to estimate triangular arbitrage opportunities between the dollar, euro, yen, sterling and Swiss franc during two-minute intervals. Estimated mean arbitrage profits, net of bid-offer spreads and 0.2 basic point transaction fees, ranged from 2.8 to 3 basis points. Reuters platform data were used in Akram, Rime and Sarno (2008) to detect covered interest arbitrage (CIP) opportunities, where the CIP no-arbitrage condition is that the forward-spot exchange rate differential matches the interest rate differentials between similar assets with the same maturity. After taking account of transaction costs, the authors discovered numerous short-lived deviations from the CIP condition that would have presented profitable arbitrage opportunities, and that the profit opportunities were large enough and of significantly long duration for traders to have exploited them with profit.

3. The Fear Index

If the financial markets are informationally efficient, as the EMH postulates, it would not be possible to design investment strategies that would consistently outperform the market. In the EMH world it is difficult to explain the excess returns of the funds managed by the likes of Soros (1995), whose early investment ventures involved arbitrage between the US and European markets, and Buffett (see Lowenstein 2001). Such funds have not always outperformed indexed portfolios, but have shown a persistent tendency to do so.

Samuelson, the co-founder of the EMH, pointed out that the EMH implies that "most portfolio decision makers should go out of business – take up plumbing, teach Greek, or help produce the annual GDP by serving as corporate executives... even if this advice to drop dead is good advice, it is obviously not counsel that will be eagerly followed" (1974, quoted in Bernstein 2007, p. 113). Samuelson himself did not follow the EMH counsel when, in 1970 he became a founder investor in Commodities Corporation, started by a Ph.D. graduate from Samuelson's MIT. This fund used statistical models of supply and demand for commodities, collecting detailed information on weather conditions affecting cocoa harvests, for example, to forecast commodity prices (Mallaby 2010, Ch. 3). As Samuelson later explained: "Fama's theory of the random walk and mine are not the same.... mine is that there are no easy pickings.... if you read the numerous papers I have written on the efficient markets hypothesis, you will realise it is not a dogma.... if you get information early, before it is widespread,

you can't help but get very rich" (Samuelson, February 5, 2008, interview quoted in Mallaby 2010, p. 418).

If the EMH were true, and there were no persistent arbitrage profits to be made in financial markets such as those for FX, it is difficult to explain the existence of "quants", mathematicians and physicists employed by financial institutions to detect patterns in the processes determining financial asset prices. The "quants" have formalised in algorithms the trends or tendencies that "chartists" tried to identify before the advent of powerful computers. A battalion of "quants" form the workforce in the hedge fund described in the novel *The Fear Index* (Harris 2011). This fund, *Hoffmann Investment Technologies*, has been set up by Alex Hoffmann, funded by a partnership with a former investment banker, Hugo Quarry. Hoffmann has a Ph.D. in physics from Princeton, and was employed on the Large Electron-Positron Collider project, the forerunner to the Large Hadron Collider, at CERN in Geneva, leaving under a cloud.

The VIXAL-4 fund operated by *Hoffmann Investment Technologies* has an investment strategy based on algorithms that identify patterns in the determination of asset prices, and also on algorithms to execute trades. A first innovation in the VIXAL-4 strategy is to "correlate recent market fluctuations with the frequency rate of fear-related words in the media – terror, alarm, panic, horror, dismay, dread, scare, anthrax, nuclear" (Harris 2011, p. 97). Trades are geared to the volatility index VIX, a measure of volatility in the Standard and Poor 500 index of US equity prices, tradable on the Chicago Board of Exchange. The second innovation is to allow the algorithmic trading strategy to evolve as new data emerges, as the market impact of the trades made is observed, and as the profit and losses associated with particular trades are revealed. Hence the evolution from VIXAL-1 to VIXAL-4 in the machine-learning algorithmic trading strategy. There is a Darwinian theme, the innovative use of fear indices being depicted through the lens of Darwin's *The Expression of the Emotions in Man and Animals* (1872); and the use of evolutionary adaptation reflecting Darwin's *On the Origin of the Species by Means of Natural Selection* (1859).

Reading a novel raises the problem of distinguishing the factual basis from the fiction. In relation to FX arbitrage processes an intriguing question is whether the trading processes described in The Fear Index give a closer insight into what happens in actual FX markets than does the vision of this world provided by the EMH. In *The Fear Index* emotions play a key role in driving behaviour on financial markets, whereas in the EMH behaviour is driven by reason alone. In the novel the trading strategy algorithm adapts to changes in the market environment, whereas in the EMH there is no incentive to search for fitter trading strategies.

The notion of rational economic man that underlies the description of financial markets provided by the EMH is difficult to square with much of the language used to describe such markets: market crashes, panic selling, herding, bull markets, bear markets, fear, market sentiment, optimism, pessimism and so on. The psychology literature is replete with examples of cognitive biases in the heuristics used by people when asked to trade on experimental markets, such as risk aversion, anchoring, framing, representativeness and over-confidence (Kahneman 2011). The neuroscience literature suggests that the amygdala, a part of the brain that plays a key role when there is a need to take account of both affective and cognitive responses when making decisions, is highly active when financial trading decisions are being made (Lo 2011). Sweat samples taken from traders on a London trading floor indicated that higher testosterone levels predicted greater profitability later in the trading day, and that cortisol levels were positively correlated with market volatility (Coates and Herbert 2008). When financial trading can be driven by emotions, the question arises of how the traders react to the profits or losses made. The adaptive markets hypothesis (Lo 2004) postulates that trading strategies evolve, with the "fitter" ones showing higher survival propensity. This is not too far from the trading world depicted in the early stages of *The Fear Index* – before the VIXAL-4 fund finds its Nemesis.

4. FX Trading Sequences

In what follows we consider a particular type of *classical arbitrage* strategy that operates on discrepancies between the exchange rates for currency pairs and their cross-exchange rates. This means that we do not consider other *classical arbitrage* strategies, such as that based on deviations from the covered interest parity (CIP) condition. Nor do we consider *latency, liquidity imbalance* or *complex event processing* arbitrage strategies. The focus is on the different combinations of arbitrage

sequences that can be pursued, and on whether the pursuit of arbitrage transactions that are active, in the sense that they are profitable, leads to a no-arbitrage state in which there are no active arbitrage opportunities. The following exposition is based on Cross, Kozyakin, O'Callaghan, Pokrovskii and Pokrovskiy (2012).

Three Currencies

In a three-currency world arbitrage operations are reasonably straightforward. Let the currencies be the US dollar ($), euro (€) and sterling (£), and their exchange rates (r) be defined as the number of units of domestic currency required to buy (or sell – we ignore the bid-ask spread between exchange rates and transaction costs in what follows) one unit of foreign currency. So, for example $r_{\$€}$, is the number of dollars required to buy one euro. Thus, ignoring reciprocal exchange rates, the principal exchange rates are the triplet:

$$(r_{\$€}, \ r_{\$£}, \ r_{€£}) \tag{1}$$

Let there be three currency traders who, in line with the "home currency bias" evidence mentioned earlier in this paper, initially are aware only of the exchange rates involving their home currency. So, the dollar trader does not initially know the value of $r_{€£}$, the euro trader is initially unaware of $r_{\$£}$ and the sterling trader is initially unaware of $r_{\$€}$.

The condition for the dollar trader to conduct profitable arbitrage, when discovering that the euro-sterling exchange rate is out of line with the dollar exchange rate, is:

$$r_{\$£} \cdot r_{£€} > r_{\$€}. \tag{2}$$

The dollar trader would then make an arbitrage profit by exchanging dollars for pounds and using the pounds to buy euros. This would continue until the new non-arbitrage exchange rate is reached:

$$r_{\$€}^{(new)} = \frac{r_{\$£}}{r_{€£}}, \ r_{\$£}, \ r_{€£}. \tag{3}$$

The euro trader would have an active arbitrage opportunity if he discovered that the dollar-sterling rate was such that:

$$r_{€\$} \cdot r_{\$£} > r_{€£}. \tag{4}$$

Here the euro trader would profit by exchanging euros for dollars and the dollars for sterling until a new no-arbitrage ensemble of exchange rate is established:

$$r_{\$€}, \ r_{\$£}, \ r_{€£}^{(new)} = \frac{r_{\$£}}{r_{\$€}}. \tag{5}$$

Finally, the sterling trader could discover that the dollar-euro exchange rate is such that:

$$r_{£€} \cdot r_{€\$} > r_{£\$}. \tag{6}$$

Here the sterling trader would profit by exchanging sterling for euros and the euros for dollars until a new no-arbitrage ensemble emerges:

$$r_{\$€}, \ r_{\$£}^{(new)} = r_{\$€} \cdot r_{€£}, \ r_{€£}. \tag{7}$$

In this three-currency world the order in which traders find information about active arbitrage opportunities matters, but one arbitrage operation, perhaps executed algorithmically to minimise market impact, would suffice to exploit and, by doing so, remove the profitable arbitrage opportunity. This is not too far from the EMH world in which active arbitrage opportunities are, at most, fleeting. The interesting question is whether this state of affairs pertains when a larger number of currencies are present.

Four Currencies

Let the fourth currency be the Japanese yen (¥). Then we have C_4^2 principal exchange rates with R indicating this ensemble:

$$R = (r_{\$€}, r_{\$£}, r_{\$¥}, r_{€£}, r_{€¥}, r_{£¥}). \tag{8}$$

The balanced set of exchange rates, in which no arbitrage opportunities exist, can be shown by inspection, to be:

$$r_{€£} = \frac{r_{\$£}}{r_{\$€}}, \quad r_{€¥} = \frac{r_{\$¥}}{r_{\$€}}, \quad r_{£¥} = \frac{r_{\$¥}}{r_{\$£}}. \tag{9}$$

Again, it is assumed that each trader is initially aware only of the exchange rates for his home currency. So, for example, the dollar trader knows $r_{\$¥}$, $r_{\$£}$ and $r_{\$¥}$ but not $r_{€£}$, $r_{€¥}$ and $r_{£¥}$.

The crucial issue in this four-currency world is the order in which traders discover information about profitable arbitrage opportunities. So, for example, if the first information about a cross-exchange rate arbitrage opportunity arrives in the form of the dollar trader finding out that the euro-sterling exchange rate is out of line with the rates for the dollar-euro and dollar-sterling currency pairs, then a dollar-euro-sterling ($€£) arbitrage sequence will be exploited first. After the exploitation of this triangular arbitrage opportunity, which is denoted as $A_{\$€£}$, a new ensemble of principal exchange rates would emerge:

$$R^{(new)} = RA_{\$€£}^{(new)} = \left(r_{\$€}^{(new)}, r_{\$£}, r_{\$¥}, r_{€£}, r_{€¥}, r_{£¥} \right). \tag{10}$$

Where $RA_{\$€£}^{(new)}$ indicates the new ensemble of exchange rates after an active arbitrage $A_{\$€£}$, has been exploited. This new exchange rate ensemble would present new cross-exchange rate arbitrage opportunities to other traders. So the crucial difference between the three and four-currency case is that in the latter case a single arbitrage opportunity does not bring the FX market into a balanced ensemble of exchange rates in which no active arbitrage opportunities exist. The exploitation of one arbitrage opportunity has a "ripple" effect, the change in cross-exchange rates creating further arbitrage opportunities about which the other traders could become informed.

The number of possible triangular arbitrage operations possible in this four-currency case is $P_4^3 = 24$. These possible arbitrage operations are listed in Table 1, along with their activation conditions and the new exchange rates arising from their exploitation:

Table 1: Lists of Arbitrages

Number	Arbitrage	Activation condition	Actions
1	$A_{\$€£}$	$r_{€£} > r_{\$€} \cdot r_{\$£}$	$r_{\$€}^{(new)} = r_{\$£}/r_{€£}$
2	$A_{\$€¥}$	$r_{\$¥} > r_{\$€} \cdot r_{€¥}$	$r_{\$€}^{(new)} = r_{\$¥}/r_{€¥}$
3	$A_{\$£€}$	$r_{\$€} \cdot r_{€£} > r_{\$£}$	$r_{\$£}^{(new)} = r_{\$€} \cdot r_{€£}$
4	$A_{\$£¥}$	$r_{\$¥} > r_{\$£} \cdot r_{£¥}$	$r_{\$£}^{(new)} = r_{\$¥}/r_{£¥}$
5	$A_{\$¥€}$	$r_{\$€} \cdot r_{€¥} > r_{\$¥}$	$r_{\$¥}^{(new)} = r_{\$€} \cdot r_{€¥}$
6	$A_{\$¥£}$	$r_{\$£} \cdot r_{£¥} > r_{\$¥}$	$r_{\$¥}^{(new)} = r_{\$£} \cdot r_{£¥}$
7	$A_{€\$£}$	$r_{\$£} < r_{\$€} \cdot r_{€£}$	$r_{\$€}^{(new)} = r_{\$£}/r_{€£}$
8	$A_{€\$¥}$	$r_{\$¥} < r_{\$€} \cdot r_{€¥}$	$r_{\$€}^{(new)} = r_{\$¥}/r_{€¥}$

9	$A_{€£\$}$	$r_{\$£} > r_{€£} \cdot r_{\$€}$	$r_{€£}^{(new)} = r_{\$£}/r_{€€}$
10	$A_{€£¥}$	$r_{€¥} > r_{€£} \cdot r_{£¥}$	$r_{€£}^{(new)} = r_{€¥}/r_{£¥}$
11	$A_{€¥\$}$	$r_{\$¥} > r_{€¥} \cdot r_{\$€}$	$r_{€¥}^{(new)} = r_{\$¥}/r_{\$€}$
12	$A_{€¥£}$	$r_{€£} \cdot r_{£¥} > r_{€¥}$	$r_{€¥}^{(new)} = r_{€£} \cdot r_{£¥}$
13	$A_{£\$€}$	$r_{\$€} \cdot r_{€£} < r_{\$£}$	$r_{\$£}^{(new)} = r_{\$€} \cdot r_{€£}$
14	$A_{£\$¥}$	$r_{\$¥} < r_{€£} \cdot r_{£¥}$	$r_{\$£}^{(new)} = r_{\$¥}/r_{£¥}$
15	$A_{£€\$}$	$r_{\$£} < r_{€£} \cdot r_{\$€}$	$r_{€£}^{(new)} = r_{\$£}/r_{\$€}$
16	$A_{£€¥}$	$r_{€¥} < r_{€£} \cdot r_{£¥}$	$r_{€£}^{(new)} = r_{€¥}/r_{£¥}$
17	$A_{£¥\$}$	$r_{\$¥} > r_{£¥} \cdot r_{\$£}$	$r_{£¥}^{(new)} = r_{\$¥}/r_{\$£}$
18	$A_{£¥€}$	$r_{€¥} > r_{£¥} \cdot r_{€£}$	$r_{£¥}^{(new)} = r_{€¥}/r_{€£}$
19	$A_{¥\$€}$	$r_{\$€} \cdot r_{€¥} < r_{\$¥}$	$r_{\$¥}^{(new)} = r_{\$€} \cdot r_{€¥}$
20	$A_{¥\$£}$	$r_{\$£} \cdot r_{£¥} < r_{\$¥}$	$r_{\$¥}^{(new)} = r_{\$£} \cdot r_{£¥}$
21	$A_{¥€\$}$	$r_{\$¥} < r_{€¥} \cdot r_{\$€}$	$r_{€¥}^{(new)} = r_{\$¥}/r_{\$€}$
22	$A_{¥€£}$	$r_{€£} \cdot r_{£¥} < r_{€¥}$	$r_{€¥}^{(new)} = r_{€£} \cdot r_{£¥}$
23	$A_{¥£\$}$	$r_{\$¥} < r_{£¥} \cdot r_{\$£}$	$r_{£¥}^{(new)} = r_{\$¥}/r_{\$£}$
24	$A_{¥£€}$	$r_{€¥} < r_{£¥} \cdot r_{€£}$	$r_{£¥}^{(new)} = r_{€¥}/r_{€£}$

Source: Cross, Kozyakin, O'Callaghan, Pokrovskii and Pokrovskiy (2012).

The key question, then, is whether the "ripple" effects arising from the exploitation of an active arbitrage opportunity results in a sequence of active arbitrages that converge onto a no-arbitrage ensemble of exchange rates. Cross et. al. (2012) apply asynchronous systems analysis to investigate the way active arbitrage chains evolve. The key finding is that "rather than there being a smooth convergence to an ensemble of exchange rates with no arbitrage opportunities, the arbitrage operations may display periodicity and no necessary convergence on a cross-exchange rate law of one price" (Cross et. al. 2012, p. 252). This periodicity finding is interesting in several respects. It suggests that active arbitrage opportunities will continue to exist in markets such as those for FX. Rather than the exploitation of active arbitrage opportunities leading to their elimination, their exploitation is followed by "ripple " effects that do not necessarily tend to fade away, new arbitrage opportunities being activated.

Preliminary results (Cross and Kozyakin 2012) suggest that triangular arbitrage in a world of five or more currencies and traders involves more unstable behaviour in arbitrage chains than in the four-currency case. In this higher currency-dimension world, exchange rates not only may display the periodicity arising in the four-currency case, "but also, what is surprising, may grow in accordance with the double exponential law" (Cross and Kozyakin 2012, p. 11). For instance, in the five-currency case exchange rates may grow as $e^{c \cdot 1.071^n}$, where n is the number of arbitrage operations and c is a positive constant. When the number of currencies and currency-traders involved is greater than five then we face an effect of "inheritance of instability" taking place in the cases of four and five currencies. In this case one may observe all the types of behaviour of exchange rates described above: periodicity, growth with the exponential rate and growth in accordance with the double exponential law.

5. Concluding Remarks

The EMH postulates that financial markets are informationally efficient, so that price movements are essentially random. Applied to FX markets, the EMH implies that profitable arbitrage opportunities are, at most, short-lived. This paper has documented the fragmented nature of FX markets, the institutional fragmentation implying that the information about profitable arbitrage opportunities available to FX participants is also fragmented. Even on single electronic trading platforms there is evidence from historical data that triangular arbitrage operations would have been profitable, net of bid-ask spreads and transaction costs.

In the world described by the EMH it is difficult to explain the existence of funds, such as the hedge fund in *The Fear Index* (Harris 2011), that employ "quants" to design algorithms to detect arbitrage opportunities. The problem here might be that the academic literature has failed to analyse the complexity of the combinations of possible arbitrage sequences in markets such as those for FX. The combinatorial analysis of such sequences is straightforward in a three-currency world. As shown in Cross, Kozyakin, O'Callaghan, Pokrovskii and Pokrovskiy (2012), in a four-currency world profitable arbitrage sequences tend to display periodicity. This finding helps to explain why arbitrage operations in markets such as those for FX are endemic rather than self-extinguishing.

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