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ON PUBLIC INEFFICIENCIES IN A MIXED DUOPOLY

BY

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On Public Inefficiencies in a Mixed Duopoly

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Abstract

The aim of this paper is to investigate the welfare effect of a change in the public firms objective function in oligopoly when the government takes into account the distortionary effect of rising funds by taxation (shadow cost of public funds). We analyze the impact of a shift from welfare- to profit-maximizing behaviour of the public firm on the timing of competition by endogenizing the determination of simultaneous (Nash-Cournot) versus sequential (Stackelberg) games using the game with observable delay proposed by Hamilton and Slutsky (1990). Differently from previous work that *assumed* the timing of competition, we show that, absent efficiency gains, instructing the public firm to play as a private one never increases welfare. Moreover, even when large efficiency gains result from the shift in public firm's objective, an inefficient public firm that maximizes welfare may be preferred.

Keywords: Mixed oligopoly; Nash equilibria; Endogenous Timing; Distortionary taxes.

JEL Classification: L1; L13; L32; L33.

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1   Introduction

In the last decades of the XX century a process of liberalization and/or privatization occurred in most of the industrialized countries and, since then, public utilities are generally no longer provided by public monopolies. The motivations for this program were essentially linked to the general perception of poor performance of public monopolies and to the idea that entry of private subjects could enhance efficiency. For example, during the nineties, in Italy, in France and in UK, as in many EU countries, the public incumbent faced the entry of private competitors in many communication services. The same occurred in the production of electricity, in gas retailing and more recently in some postal services. In the same years, national (public) airlines started competing with private or foreign ones in the domestic markets. Moreover, examples of public monopolies that became mixed oligopoly can be found in a broad range of industries including railways, steel and overnight-delivery industries, as well as services including banking, home loans, health care, life insurance, hospitals, broadcasting, and education.[1] In these cases, instead of regulating a privatized monopoly, governments decided to enforce a facility-based competitions in order to achieve a so-called dynamic efficiency.[2] In other cases, even if public firms had not been privatized, it was required its management to operate satisfying the budget balance constraint or to directly maximize profits, as private firms do.

Our investigation starts downward the liberalization process of a public monopoly, and the aim of the present work is to build a theoretical model for analyzing the welfare effect of a change in its objective function in oligopoly[3]. Since we mainly refer to public utility markets open to competition, we consider a mixed duopoly in which firms are characterized by increasing returns to scale (with fixed and constant marginal costs) and we assume that the public firm is *typically* less efficient than its private competitor.[4]

The first novel contribution of this paper is represented by the introduction of the shadow cost of public funds in the public firm's objective function. That is, we assume that the public firm is required to take into account the distortionary effect of the taxes that are needed to cover its deficit and, in general, public expenditures. In fact, absent lump-sum tax instruments, if government rises 1 Euro from taxation, society pays $(1 + \lambda)$ Euros. Coherently, public profits, when positive, avoid an equivalent public transfer, reducing distortionary taxes.[5] As initially analyzed in Meade (1944) and exploited in Laffont and Tirole (1986, 1993), this approach has been used to characterize public monopolies running a deficit and, more generally, regulated markets. Here we apply the same analysis to a public firm competing in a duopoly and to the effects of privatization or of a change f public firm' objective function like profit

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[1]In industrial organization the term *mixed oligopolies* has been used to describe imperfectly competitive markets in which public firms compete with private ones.

[2]For deeper viewpoints on the role played by facility-based competition in EU and US Telecommunications liberalization and regulation processes see Taschdjian (1997) and Stehmann and Borthwick (1994).

[3]This change can be the consequence not only of a privatization process but also of a political choice.

[4]Differently from Cremer et al. (1989), the public firm's higher cost is not a neutral transfer from firm to workers belonging to the same economy but, as an $X-$inefficiency, it reduces any utilitarian measure of welfare. Moreover, we can represent with an extra cost any problem of controlling and/or monitoring that derive from agency problem that affect public firms but that we don't analyze. Notice that we do not consider that in any context public firms are less efficient than private competitors, but we start from the worst context and we prove that inefficiency cannot matter.

[5]Since public firm's profit or deficit are not a neutral transfer among agents of the same economy, they ought not to be weighted as private firm's profits or consumer net surplus in the utilitarian measure of welfare, but they should be weighted $(1 + \lambda)$.

maximization, given that *getting money for reducing public debt or distortionary taxes*, is often a complementary target[6]. The main consequence is that, taking into account the shadow cost of public funds, the public firm puts more weight on its own profits mimicking, at least partially, the behaviour of a private firm.

The second contribution of this work is that we consider the effect of a change in the public firm's objective function on the timing of competition by endogenizing the determination of simultaneous (Nash-Cournot) versus sequential (Stackelberg) games. That is, the structure of the game is not assumed *a priori*, but is the result of preplay independent and simultaneous decisions by the players. In fact, in many economic situations it is often more reasonable to assume that firms choose not only what action to take, but also when to take it. Moreover, we believe that this approach is especially relevant for the analysis of privatization, given that results and policy prescriptions emerged in the literature crucially rely on the type of competition *assumed*. For example, in de Fraja and Delbono (1989) it is shown that privatization may improve welfare under Cournot competition even without efficiency gains; while, if a Stackelberg game with public leadership is exogenously assumed, this cannot occur.[7] In another paper, Beato and Mas-Colell (1984) show that welfare may be higher when the public firm is the follower than when it is the leader in a Stackelberg game. In this way they provided an argument against the standard view of the so-called Second-Best literature (see, for example, Rees, 1984; Bös, 1986) that claimed the sub-optimality of the marginal-cost pricing rule.[8]

In the present work, in order to endogenize the timing of the game, we apply the model developed by Hamilton and Slutsky (1990) to the mixed oligopoly framework. In their insightful paper, the authors build an endogenous timing game by adding to the basic quantity game a preplay stage at which players simultaneously and independently decide whether to move early or late in the basic game. Therefore, the type of competition endogenously emerges in the subgame-perfect equilibrium (SPE) of this extended game. Amir and Grilo (1999) apply this model to a private duopoly showing that, in a quantity setting with strategic substitutability, Cournot equilibria always result as the SPE of the endogenous timing game. Pal (1998) addresses the issue of endogenous order of moves in a mixed oligopoly by adopting the same game structure. It is shown that sequential playing always emerges as the endogenous timing and both Stackelberg solutions are the SPE of the mixed-duopoly game. Even though after Pal (1998) other authors analyzed the endogenous timing in mixed oligopolies, there is no work, at our best knowledge, that extends this line of research to the welfare evaluation of privatization.[9]

The main results of our analysis can be summed up as follows.

With respect to the determination of the endogenous timing in mixed oligopoly, our results differ from Pal (1998), since in our model setting either Nash, or private leadership or both Stackelberg outcomes can result as subgame-perfect equilibria (SPE) of the endogenous timing game. Moreover, following the intuition of Beato and Mas-Colell (1984), we show that when both Stackelberg game are SPE of the endogenous timing game, private leadership is preferred

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[6]Given our model setting, it will be clear than there is no differences between to privatize a public firm and to ask its management to maximize no welfare but profits. This case is what we call a change in the public firm's objective function.

[7]The assumption of decreasing returns to scale is fundamental to their result that privatization may increases welfare. This is generally not the case of a public utility provider.

[8]See de Fraja and Delbono (1990) for a survey of these models.

[9]Matsumura (2003), Cornes and Sepahvand (2003) and Sepahvand (2004) apply the same model to international mixed oligopolies finding that public leadership may emerge as the unique SPE of the endogenous timing game.

by the public firm and it is indeed selected when risk-dominance is used as the equilibrium selection criterion.

This result is crucial to the evaluation of the welfare impact of asking a public firm to maximize profits. In fact, differently from de Fraja and Delbono (1989), absent efficiency gains, profit maximization never increases welfare. Furthermore, even when large efficiency gains are realized, an inefficient public firm that maximize welfare may still be preferred. The last result relies on the fact that only with a public firm sequential outcomes (that are always welfare superior) may be supported as SPE of the endogenous timing game. Conversely, with private-owned firms, simultaneous equilibria are always implemented.

It is worth noting that our results are obtained in a context of complete information, and under the assumption that government has the full bargaining power in the privatization process; that is, the price paid by the new private owners for the former public firm is assumed to be equal to its profit in the new (Cournot) equilibrium. This assumption drives the results in favour of privatization or of asking public firms to maximize profits, since it overweights the profits by λ in any welfare comparison.

In what follows, the next Section sets up the model. Section 3 is focused on the issue of endogenous timing in mixed oligopoly, while Section 4 is devoted to the analysis of a change of the public firm's objective function. Our conclusions are delegated to Section 5. All the proofs are collected in the Appendix.

2 The basic setting

In a static, partial equilibrium analysis, we consider the simplest setting of a mixed duopoly, where a private and a public firm, labelled with $i = p, g$ respectively, produce a commodity and compete in a quantity game. Demand preferences are described by a linear function where intercept and slope are normalized to one:

$$p(q_g, q_p) = 1 - q_p - q_g$$

Both firms are characterized by constant marginal costs, $c_i \geq 0$, and fixed costs, $K_i \geq 0$ [10]. We assume that the public firm is already in the market and its fixed cost is sunk. Conversely, the private firm's fixed cost is borne only in case of producing. Moreover, the private firm's marginal cost is normalized to zero, $c_p = 0$, while the public firm's one is positive, $c_g = c \geq 0$. That means, c is an index of the public firm's inefficiency.

The private firm maximizes its profit:

$$\Pi_p(q_g, q_p) = (1 - q_g - q_p) q_p - K_p$$

The public firm maximize a utilitarian measure of welfare taking into account the shadow cost of public funds, $\lambda > 0$. This parameter is a measure of the dead-weight loss due to distortionary taxation. In particular, let $S(Q)$ denote the consumer gross surplus, where $Q = q_p + q_g$ is the industry total output. We assume that government can choose the public firm's output level

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[10]We consider that the assumption of increasing returns to scale is coherent with the presence of a public incumbent, former monopolist, in a liberalized public utility industry. Nevertheless, the assumption decreasing return to scale is popular in the literature. For papers adopting constant marginal costs, see Cremer et al. (1989) and Martin (2004); while for papers adopting increasing marginal costs, see Beato and Mas-Colell (1984), de Fraja and Delbono (1989), Fjell and Pal (1996), Pal and White (1998), Matsumura (1998).

q_g and it can make a monetary transfer T to the public firm. Then, in the presence of the shadow cost of public funds, the maximization problem of the government is:

$$\max_{q_g, T} \quad W(q_p, q_g) = S(q_p + q_g) - C_g(q_g) - C_p(q_p) - \lambda T$$

$$\text{such that} \quad \widetilde{\Pi}_g = p(q_p + q_g)q_g - C_g(q_g) - K_g + T \geq 0 \tag{1}$$

where $\widetilde{\Pi}_g$ is the public firm's budget including the (positive or negative) transfer T. Notice that the (participation) constraint (1) is not a hard budget balance constraint but it is compatible with operative losses when T is positive. From (1) we get

$$T = \widetilde{\Pi}_g - [p(Q)q_g - C_g(q_g) - K_g]$$

and substituting T in the objective function we obtain:

$$
\begin{aligned}
\max_{q_g} \quad W &= S(q_p + q_g) - C_g(q_g) - C_p(q_p) - \lambda \left[\widetilde{\Pi}_g - (p(Q)q_g - C_g(q_g) - K_g) \right] \\
&= S(q_p + q_g) - C_g(q_g) - C_p(q_p) + \lambda \left(p(Q)q_g - C_g(q_g) - K_g \right) - \lambda \widetilde{\Pi}_g \\
&\quad \text{such that} \quad \widetilde{\Pi}_g \geq 0
\end{aligned}
$$

Since welfare is decreasing in $\widetilde{\Pi}_g$ when λ is positive, it is optimal to set $\widetilde{\Pi}_g = 0$. Then, we have:

$$T = - \left[p(Q)q_g - C_g(q_g) - K_g \right]$$

The problem can be rewritten unconstrained as follows:

$$\max_{q_g} \quad W = S(q_p + q_g) - C_g(q_g) - C_p(q_p) + \lambda \left(p(Q)q_g - C_g(q_g) - K_g \right)$$

Defining the consumer net surplus as

$$CS(Q) = S(Q) - p(Q)(q_p + q_g) \tag{2}$$

and the public firm's operative profit as

$$\Pi_g(q_g, q_p) = p(Q)q_g - C_g(q_g) - K_g \tag{3}$$

the maximization problem of the government is reduced to:

$$\max_{q_g} \quad W(q_g, q_p) = CS(Q) + \Pi_p(q_g, q_p) + (1 + \lambda)\Pi_g(q_g, q_p) \tag{4}$$

So, the objective defined in equation (4) implies that a transfer occurs in order to guarantee the public firm's budget balance. This transfer is positive (negative) when the public firm's profits are negative (positive).

The objective function (4) can be also interpreted as a weighted average of welfare, defined as the net surplus generated in the market, and the public firm's profit, where the former is weighted by $1/(1 + \lambda)$, the latter by $\lambda/(1 + \lambda)$.

$$
\begin{aligned}
W(q_g, q_p) &= V(q_g, q_p) + \lambda \Pi_g(q_g, q_p) \tag{5} \\
&> \frac{1}{1 + \lambda} V(q_g, q_p) + \frac{\lambda}{1 + \lambda} \Pi_g(q_g, q_p)
\end{aligned}
$$

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$$where \quad V(q_g, q_p) = CS(Q) + \Pi_g(q_g, q_p) + \Pi_p(q_g, q_p)$$

We know that other works on mixed oligopoly consider a weighted average of welfare as public firm's objective function. In fact, if we had assumed a hard budget balance constraint without a public transfer, as in Cremer et al. (1989), the weight given to the public firm's profit would have been endogenous and equal to the associate Lagrangian multiplier. Alternatively, as in Hindriks and Claude (2006) the weight could be positively related to the endogenous share of a partially privatized firm owned by private investors, while as in Matsumura (1998), due to incentive problem between government and public management, in equilibrium a negative relation may occur. In the present paper the weight of the public firm's profit is exogenously correlated to the shadow costs of public funds. In our analysis, introducing λ extends the contract theory approach of public monopoly regulation to the case of (mixed) oligopoly.

The best-reply (or reaction) function of the private firm is derived, as usual, from the first order condition:

$$\frac{\partial \Pi_p(q_g, q_p)}{\partial q_p} = p(q_g, q_p) + p'(q_g, q_p) q_p = 0$$

In the presence of fixed costs, the private firm's reaction function ought to be truncated in the point it crosses the zero-isoprofit curve and on-the-boundary solutions can occur in equilibrium. Given the model setting, it can be written explicitly in the following way:

$$r_p(q_g) = \begin{cases} \frac{1}{2}(1 - q_g) & if \quad q_g < \overline{q_g} \\ 0 & if \quad q_g \geq \overline{q_g} \end{cases} \tag{6}$$
$$where \quad \overline{q_g} = q_g : \Pi_p(r_p(q_g), q_g) = 0$$

The public firm's first order condition can be derived from the objective (5):

$$\frac{\partial W(q_g, q_p)}{\partial q_g} = \frac{\partial V(q_g, q_p)}{\partial q_g} + \lambda \frac{\partial \Pi_g(q_g, q_p)}{\partial q_g}$$
$$= [p(q_g, q_p) - c] + \lambda [p(q_g, q_p) - c + p'(q_g, q_p) q_g] = 0$$

Notice that when $\lambda = 0$ public firm's output decision follows the marginal cost pricing rule, and the first term in square brackets measure its effect on total surplus (*allocative* effect). The second term is the effect on the public firm's profits, since the latter prevents the government from using distortionary taxation to raise money (we call it the *distortionary* effect). When $\lambda \to +\infty$, the public firm plays as a private (Cournot) competitor.

Since there is no hard budget balance constraint and its fixed cost K_g is sunk, the public firm's reaction function is not truncated and it can be explicitly derived:

$$r_g(q_p) = \max \left\{ \frac{1+\lambda}{1+2\lambda} (1 - c - q_p); 0 \right\} \tag{7}$$

However, we want to focus on the case in which both firms produce strictly positive quantities when they play simultaneously; so, we provide some assumptions on the admissible set in the parameters space.

Assumption 1 *The parameters c and λ belong to the subspace*

$$A \subset \mathbb{R} \times \mathbb{R} = \left\{ (c, \lambda) \, | c \in \left(0, \frac{1}{2}\right) \vee \lambda \in [0, \overline{\lambda}] \right\}$$

where $\overline{\lambda}$ is a finite, reasonable value of the shadow cost of public funds.

Assumption 2 *The private firm's fixed cost K_p belongs to the subspace $B \subset \mathbb{R} = \left[0, \overline{K}_p\right]$, where $\overline{K}_p$ is smaller than the producer surplus of the private firm in any (simultaneous or sequential) equilibrium.*

In Figure 1, the reaction functions are depicted. Coherently with the Assumptions 1 and 2, the intersection occurs in the interior of the parameters space where both firms produce strictly positive quantities.
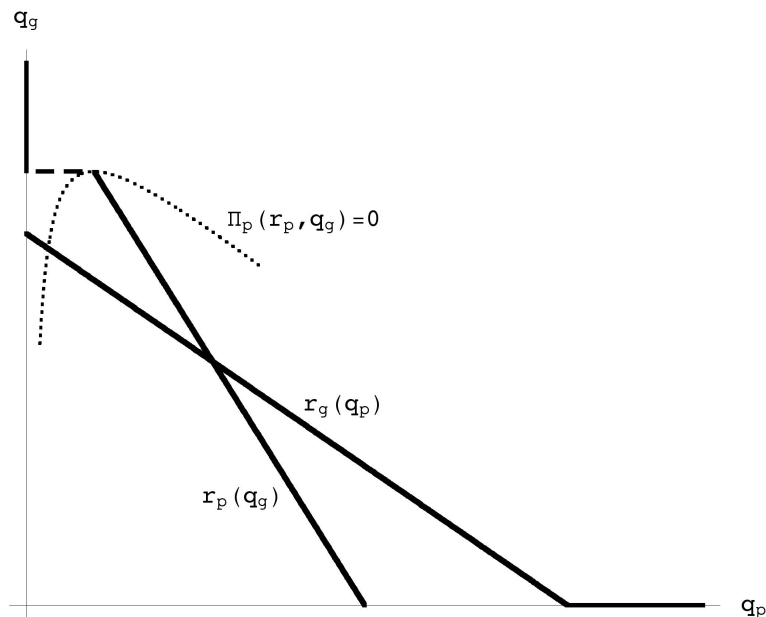

Figure 1: Firms' reaction functions

An increase in λ has the effect of reducing both intercept and slope of the public firm's best-reply. When λ tends to infinite, the public firm plays as a profit maximizer.

3   Endogenous timing in a mixed duopoly

In this Section we investigate how the determination of simultaneous (Nash-Cournot) versus sequential (Stackelberg) games is the result of preplay independent and simultaneous decisions by the players.[11]

In many mixed oligopoly and privatization works, the timing of the competition (simultaneous or sequential) is generally assumed, and simultaneous playing is mostly adopted.[12] Of course, this assumption is not neutral and it affects the results and the policy prescriptions on privatization. For example, without efficiency gains, in de Fraja and Delbono (1989) it is

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[11]Notice that in formal game-theoretical terms, Stackelberg's proposal is not to be understood as a new solution concept for one-shot games, but rather as a subgame-perfect equilibrium of a two-stage game of perfect information with exogenously given first and second movers.

[12]In Cremer et al. (1989, p. 284), the reason for using a simultaneous timing is summarized as follows: *"The common use of the Nash equilibrium in industrial organization [...] suggests that this is at least as plausible as the leader-follower situations [...]."*

shown that privatization never improves welfare when a Stackelberg game with public leadership is played; on the contrary, privatization may be welfare improving in the simultaneous setting. Then, the welfare impact of privatization crucially depends on the assumed timing.

More recently, other works introduced the idea that the order of play should result from the players' timing decision. In particular, in a private duopoly with strategic substitutability it has been proved by Hamilton and Slutsky (1990) and by Amir and Grilo (1999) that simultaneous play emerges as the unique equilibrium of the endogenous game. Conversely, in a mixed duopoly Pal (1998) shows that sequential play always occurs in equilibrium.

Coherently with this approach, in order to endogenize the timing of the play, we use the game with observable delay defined by Hamilton and Slutsky (1990). In the first stage firms simultaneously and independently choose the timing of action (whether to move *early* or *late*) and then, once observing each other decision, they play the basic quantity game. The extensive form of the game is represented in Figure 2.
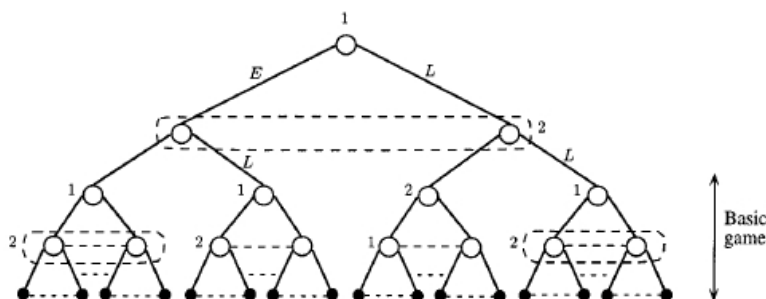

Figure 2: The extensive form of the game with observable delay (Hamilton and Slutsky, 1990)

The relevant equilibrium concept is the subgame-perfect equilibrium (SPE) and each player decides the timing of action according to the outcomes in the second stage (the basic game). Of course, none of the firms can choose the type of competition by itself, but it can only eliminate some outcome. For example, if firm i decides to move early two outcomes are possible according to the decision of the other player; only the Stackelberg outcome where firm i is follower is ruled out by its decision.

Assuming existence and uniqueness of equilibria in each basic game, the following Proposition summarizes the results obtained in Hamilton and Slutsky (1990) for any two-player game.

Proposition 3 *Consider a two-player game for which the Nash and the two Stackelberg equilibria exist. Given that both players always prefer to be a Stackelberg leader than a simultaneous player, the set of (pure strategy) SPE of the endogenous timing game is defined in the following way:*

i) if the Stackelberg follower payoff is lower than the Nash payoff for each firm, then the unique SPE of the endogenous timing game is the Nash equilibrium where both firms decide to move early;

ii) if the Stackelberg follower payoff is strictly larger than the Nash payoff for each firm, then the both Stackelberg equilibria are SPE of the endogenous timing game;

iii) if firm i's Stackelberg follower payoff is strictly larger than its Nash payoff and if firm j prefers to play simultaneously than to be Stackelberg follower, then the unique SPE of the endogenous timing game is the Stackelberg equilibrium with firm j being the leader.

Proof. The proof of this Proposition follows from Theorems II, III and IV in Hamilton and Slutsky (1990). ∎

The intuition behind these results is the following. Given that both firms prefer to be leader than to play simultaneously, if the Nash payoff is higher than the follower payoff, then any firm has a dominant strategy to move early. But if one firm prefers its follower payoff to the Nash payoff, there is no dominant strategy: when the other player moves early it prefers to move late and vice versa. This explains the three possible outcomes listed in Proposition 3.

We use Proposition 3 in order to determine the endogenous timing equilibrium, where the existence and the uniqueness of the equilibria in each basic game are assured by Assumptions 1 and 2. The reduced form of the endogenous timing game for the mixed duopoly is represented in Table 1.

	Private Firm	
Public Firm	Early	Late
Early	$W^{MN}(.),\ \Pi_p^{MN}(.)$	$W^{GL}(.),\ \Pi_p^{GL}(.)$
Late	$W^{PL}(.),\ \Pi_p^{PL}(.)$	$W^{MN}(.),\ \Pi_p^{MN}(.)$

Table 1: The reduced form of the endogenous timing game. MN, PL and GL stay for Nash, Private Leadership and Public Leadership equilibria respectively.

In order to solve the game we need to compare the equilibrium payoffs in each basic game. In what follows the simultaneous and sequential equilibria are derived.

3.1 Simultaneous equilibrium

When firms play simultaneously, the equilibrium output levels solve the system of the best-reply functions (6) and (7). We refer to this equilibrium as a *mixed duopoly Nash equilibrium* and the equilibrium values are labeled by MN. The output levels and the price in equilibrium are

$$q_g^{MN} = (1+\lambda)\frac{(1-2c)}{(3\lambda+1)}; \qquad q_p^{MN} = c + \lambda\frac{(1-2c)}{(3\lambda+1)}$$

$$Q^{MN} = (1-c) - \lambda\frac{(1-2c)}{(3\lambda+1)}; \qquad p^{MN} = c + \lambda\frac{(1-2c)}{(3\lambda+1)}$$

Notice that when $\lambda = 0$, the public firm's equilibrium output is such that the market price is always equal to its marginal cost. It means that the public firm implements a total output level equal to the one derived in the case of a welfare maximizer (but inefficient) monopoly; but now the welfare is higher.[13] Moreover, when the public firm is as efficient as the private one, the first best solution is implemented.

As λ increases, the public firm equilibrium output q_g^{MN} decreases and q_p^{MN} increases; then, the industry total cost decreases enhancing *productive efficiency*. This is because the public firm's concern for public transfers serves as a credible commitment to decrease its output. Moreover, since the best-reply functions are contractions, the total output level,

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[13]This is because the same total output is partially produced by the more efficient private competitor.

Q^{MN}, decreases and the market price p^{MN} increases. It is obvious that the effect on consumer surplus is negative, raising an *allocative inefficiency*.[14] The private firm's profit and welfare represent the payoffs of the players and in the simultaneous case are:

$$\Pi_p^{MN} = \left(\frac{c+\lambda(1+c)}{3\lambda+1}\right)^2 - K_p \tag{8}$$

$$W^{MN} = \frac{1-2c(1+\lambda)(1+2\lambda)^2+c^2(1+\lambda)^2(3+8\lambda)+2\lambda(3+\lambda(5+\lambda))}{2(3\lambda+1)^2} - (1+\lambda)K_g - K_p \tag{9}$$

3.2 Sequential equilibria

A Stackelberg equilibrium of this game corresponds to the SPE of a two stage game of perfect information in which the second mover (follower) chooses an action after having observed the action of the first mover (leader). Then, the Stackelberg equilibrium imposes that: (i) the strategy of the second mover is a selection from its reaction function; and (ii) the first mover chooses an action that maximizes its objective given the anticipation of the rival's reaction.

In what follows we first analyze the case of public leadership and then the private leadership equilibrium.

Public leadership (GL).

When the public firm moves before its private competitor, the equilibrium quantities solve the following equation system:

$$q_g^{GL} = \arg\max W\left(q_g, r_p(q_g)\right)$$
$$q_p^{GL} = r_p(q_g^{GL})$$

The solution is:

$$q_g^{GL} = \max\left\{\frac{(1+2\lambda)-4c(1+\lambda)}{(1+4\lambda)}, 0\right\}$$
$$q_p^{GL} = \frac{1}{2}\left(1-q_g^{GL}\right)$$

We have to distinguish two cases since there exists a threshold value of the marginal cost of the public firm such that $\forall c \in \left(0, \frac{1+2\lambda}{4(1+\lambda)}\right)$ the public firm produces a positive quantity in equilibrium. When $c \in \left[\frac{1+2\lambda}{4(1+\lambda)}, \frac{1}{2}\right)$, the public firm prefers not to produce and the private firm acts as a monopolist: its quantity, market price, and welfare are the same as in a private monopoly.

Since the threshold value $\frac{1+2\lambda}{4(1+\lambda)}$ is increasing, as λ increases an higher level of inefficiency is compatible with positive production by the public firm. In Table 2 quantities, profits and welfare in the public leadership equilibrium are summarized.

Private leadership (PL).

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[14]There exists a clear trade off between technical and allocative efficiency, and the net effect on total surplus is ambiguous and depends on the parameters.

	q_p^{GL}	q_g^{GL}	Π_p^{GL}
$c < \frac{1+2\lambda}{4(1+\lambda)}$	$\frac{2c(1+\lambda)+\lambda}{(1+4\lambda)}$	$\frac{(1+2\lambda)-4c(1+\lambda)}{(1+4\lambda)}$	$\frac{(2c(1+\lambda)+\lambda)^2}{(1+4\lambda)^2} - K_p$
$c \geq \frac{1+2\lambda}{4(1+\lambda)}$	$\frac{1}{2}$	0	$\frac{1}{4} - K_p$
	W^{GL}		
$c < \frac{1+2\lambda}{4(1+\lambda)}$	$\frac{1}{2}\frac{\left(4\lambda-2c-6c\lambda+4c^2+\lambda^2-4c\lambda^2+8c^2\lambda+4c^2\lambda^2+1\right)}{(1+4\lambda)} - (1+\lambda)K_g - K_p$		
$c \geq \frac{1+2\lambda}{4(1+\lambda)}$	$\frac{3}{8} - (1+\lambda)K_g - K_p$		

Table 2: The public leadership (GL) equilibrium quantities, profits and welfare.

	q_p^{PL}	q_g^{PL}	Π_p^{PL}
$c < \frac{\lambda}{3\lambda+1}$	$\frac{1}{2}\frac{(c+\lambda+c\lambda)}{\lambda}$	$\frac{(\lambda-c-3c\lambda)(1+\lambda)}{2\lambda(1+2\lambda)}$	$\frac{1}{4}\frac{(c+\lambda+c\lambda)^2}{\lambda(2\lambda+1)} - K_p$
$c \geq \frac{\lambda}{3\lambda+1}$	$1-c$	0	$c(1-c) - K_p$
	W^{PL}		
$c < \frac{\lambda}{3\lambda+1}$	$\frac{\left(4\lambda-4c\lambda+4c^2+8\lambda^2+\lambda^3-10c\lambda^2+17c^2\lambda-6c\lambda^3+22c^2\lambda^2+9c^2\lambda^3\right)}{8(2\lambda+1)\lambda} - (1+\lambda)K_g - K_p$		
$c \geq \frac{\lambda}{3\lambda+1}$	$\frac{1}{2} - \frac{1}{2}c^2 - (1+\lambda)K_g - K_p$		

Table 3: The private leadership (PL) equilibrium values of quantities, profits and welfare.

Assume that the private firm moves before its public competitor, that is, it behaves as a leader in the Stackelberg game. The equilibrium quantities solve the following equation system:

$$q_p^{PL} = \arg\max \Pi_p \left(r_g(q_p), q_p \right)$$
$$q_g^{PL} = r_g(q_p^{PL})$$

The solution is:

$$q_p^{PL} = \min\left\{ \frac{1}{2}\frac{(c+\lambda+c\lambda)}{\lambda}, 1-c \right\}$$
$$q_g^{PL} = \frac{1+\lambda}{1+2\lambda}\left(1-c-q_p^{PL}\right)$$

As before, we have two different cases depending on the value of c. $\forall c \in \left(0, \frac{\lambda}{3\lambda+1}\right)$ the public firm produces a positive quantity in equilibrium; more precisely, it is optimal for the private leader to choose a quantity such that the public firm's best response is positive. When $c \in \left[\frac{\lambda}{3\lambda+1}, \frac{1}{2}\right)$ the public firm does not produce in equilibrium and the private firm plays as a public (inefficient) monopolist: its quantity, as a limit level, is such that the market price is equal to the marginal cost of the public firm.[15] Of course total surplus is higher because the private competitor produces more efficiently.

Moreover, as λ increases, a larger inefficiency is compatible with a positive production by the public firm. In Table 3 quantities, profits and welfare in the private leadership equilibrium are summarized.

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[15]This is the standard case when $\lambda = 0$: the public follower can always produce the quantity needed to achieve this target and, anticipating this strategy, the best action for the private firm is to produce that quantity.

3.3 Endogenous timing equilibria

In this section we derive the endogenous timing equilibria of the mixed duopoly game. In order to apply Proposition 3 we need to rank the private and public firms' payoff in the different equilibria. In particular, in Lemma 4 we compare the private firm's profit under public leadership (i.e., the follower payoff) and in the Nash equilibrium, while in Lemma 5 we compare welfare under private leadership (again the follower payoff) with the one in the Nash equilibrium. It is worth noting that these comparisons are sufficient to apply Proposition 3. In fact, any player always prefers to be leader than to play simultaneously, by the nature of Stackelberg equilibria. Moreover, the comparison between the leader and the follower payoff is useless since no firm can unilaterally switch from one sequential equilibrium to the other.

Lemma 4 *There exists a subspace $F_1 = (c,\lambda) \subseteq A$, such that the private firm strictly prefers the public leadership equilibrium to the mixed duopoly Nash equilibrium. In the subspace $\widehat{F}_1 = A - F_1$ the reverse is true.*

This result totally relies on the choice of the public leader to produce more or less than in the simultaneous equilibrium; and this choice depends on the public firm's objective being increasing or decreasing in the rival's output in the Nash equilibrium point.

In fact, private firm's profit is strictly decreasing in the public firm's output in any interior point[16], and so, if $q_g^{PL} < q_g^{MN}$, the private firm prefers to be follower than to play simultaneously.

The public leader chooses to produce a smaller quantity with respect to the Nash equilibrium if $\frac{\partial W(q_g,q_p)}{\partial q_p} > 0$ in the Nash equilibrium. In fact, if its objective is increasing in the quantity produced by the rival, the public leader prefers to reduce its quantity anticipating that the private firm will increase the output, enhancing in this way the welfare.[17]

$$\frac{\partial W(q_g,q_p)}{\partial q_p} = \frac{\partial V}{\partial q_p} + \lambda \frac{\partial \Pi_g}{\partial q_p} = p(q_g,q_p) + \lambda p'(q_g,q_p) q_g = p(q_g,q_p) - \lambda q_g$$

In the Nash equilibrium:

$$
\begin{aligned}
\left. \frac{\partial W(q_g,q_p)}{\partial q_p} \right|_{\left(q_g^{MN},q_p^{MN}\right)} &= c + \lambda \frac{(1-2c)}{(3\lambda+1)} - \lambda(1+\lambda) \frac{(1-2c)}{(3\lambda+1)} \\
&= \frac{c\left(2\lambda^2 + 3\lambda + 1\right) - \lambda^2}{(3\lambda+1)}
\end{aligned}
$$

Then, $\left. \frac{\partial W(q_g,q_p)}{\partial q_p} \right|_{\left(q_g^{MN},q_p^{MN}\right)} > 0$ if $c > \frac{\lambda^2}{2\lambda^2+3\lambda+1}$.

This result occurs when the increase in productive efficiency due to the shift of some production to the private firms outweighs the negative allocative efficiency effect due to the reduction in total quantity and the negative distortionary effect due to the reduction in profits.

The threshold $\overline{c}(\lambda)$ is increasing because, as λ increases, the distortionary effect makes the public firm more willing to produce a larger quantity. So, only if c is high enough the overall effect of shifting some production to the efficient private competitor is positive.

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[16]Indeed, $\partial \Pi_p(q_g,q_p)/\partial q_g = p'(q_g,q_p) q_p < 0 \quad \forall q_p > 0$.

[17]Note that welfare increases despite the total quantity reduction.

Lemma 5 *There exists a subspace $F_2 = (c, \lambda) \subseteq A$, such that the public firm strictly prefers the private leadership equilibrium to the mixed duopoly Nash equilibrium. In the subspace $\widehat{F}_2 = A - F_2$ the reverse is true.*

As in the previous Lemma, this result has to do with the fact that $W(q_g, q_p)$ may be increasing in q_p. But now what matters is the decision of the private leader, that always increases its output with respect to the Nash equilibrium. As a result, total output increases and the allocative efficiency effect is positive. Recalling that moving from Nash to the public leadership equilibrium had a negative allocative efficiency effect, we would expect that the parameter space F_2 is larger F_1. This intuition is confirmed by the comparison of the thresholds $\underline{c}(\lambda)$ and $\overline{c}(\lambda)$. In fact,

$$\underline{c}(\lambda) = \frac{3\lambda^2 + 7\lambda^3}{21\lambda + 34\lambda^2 + 17\lambda^3 + 4} < \frac{\lambda^2}{2\lambda^2 + 3\lambda + 1} = \overline{c}(\lambda) \qquad \forall \lambda \in (0, \overline{\lambda})$$

Then, $F_1 \subset F_2$.

In the following Theorem the different SPE of the endogenous timing game are derived.

Theorem 6 *Consider a mixed duopoly game in which the order of moves is endogenous. The SPE of the endogenous timing game are defined in the following way:*

a) *When $(c, \lambda) \in \widehat{F}_2$, the mixed duopoly Nash equilibrium is the unique SPE of the endogenous timing game;*

b) *When $(c, \lambda) \in \widehat{F}_1 \cap F_2$, the unique SPE of the endogenous timing game is the Stackelberg equilibrium with the private firm acting as leader;*

c) *When $(c, \lambda) \in F_1$, both Stackelberg outcomes are the (pure strategy) SPE of the endogenous timing game.*

Figure 3 depicts the three possible outcomes of the endogenous timing game in the space (c, λ). Without considering λ, the previous literature (Pal, 1998) defines a unique solution where both sequential equilibria are SPE. The novel contribution of our analysis is to enlarge the set of possible outcomes defining conditions under which either private leadership or Nash equilibrium may arise as the unique SPE. The intuition is straightforward. Since the public firm's objective function is a weighted average of total surplus and profits, for low values of λ (given c) our result coincides with Pal's; for high values of λ (given c), the public firm mimics the private firm's behaviour and we obtain the same results as Amir and Grilo (1999) in a private duopoly. For intermediate values of λ, private leadership is the unique SPE since the public firm is more willing to accept a reduction in its own output when total quantity increases (in the PL equilibrium) than when total output decreases (in the GL equilibrium).

Moreover, when we focus on the sequential outcomes, the introduction of λ increases the level of inefficiency compatible with a strictly positive quantity produced by the public firm in equilibrium. In particular, in the private leadership case, Pal (1998) shows that the public firm never produces and its presence has a mere strategical role that induces the competitor to produce the limit quantity. In our framework, taking into account the shadow cost of public funds, the public firm usually produces a positive quantity in equilibrium.[18]

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[18]This is true as far as the public firm is not too inefficient. This result enhances the realism of our approach where the public firm represents not only a threat of producing, but it has an active role in the industry.
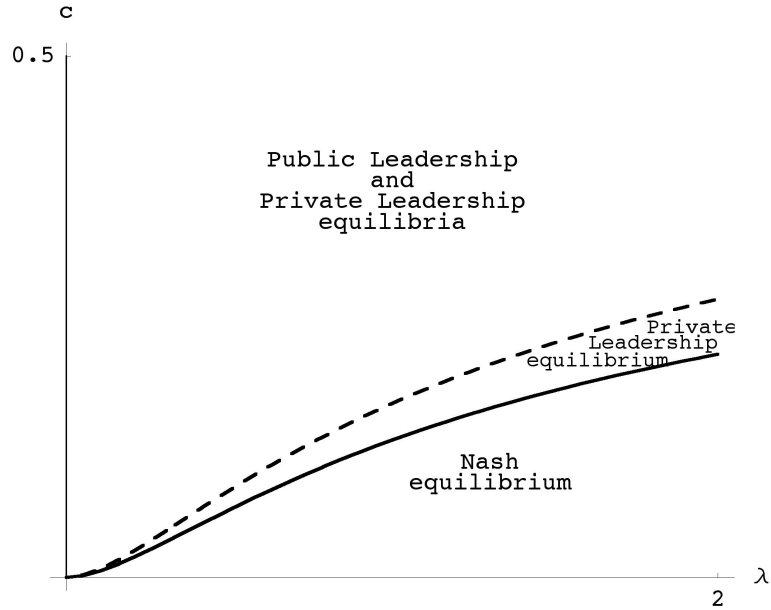
Figure 3: SPE of the endogenous timing game in the space of parameters (c, λ)

3.4 Equilibrium selection

In the subspace F_1 the endogenous timing game of the mixed duopoly has two pure-strategy Nash equilibria. Then, this is a standard coordination game with two pure-strategy Nash equilibria and one mixed-strategy equilibrium. We now analyze the pure-strategy equilibrium selection problem according to the risk dominance criterion developed by Harsanyi and Selten (1988).[19] Applied to coordination games with two pure-strategy equilibria, this procedure picks the equilibrium that has the largest basin of attraction in the initial beliefs of the players on each other's behaviour. In other words, it minimizes the risk of a coordination failure due to the strategic uncertainty that players face in a coordination game (Amir and Stepanova, 2006). This criterion proved to be a powerful selection concept in experimental settings of coordination games (Cooper et al., 1990; Van Huyck et al., 1990) and in evolutionary games characterized by experimentation and myopic learning (Ellison, 1993; Kandori et al., 1993). It is shown that the risk dominant equilibrium is often selected even when it is Pareto dominated by another equilibrium. One equilibrium risk-dominates the other if it is associated with the larger product of deviation losses. In our framework this means that private leadership is selected using the risk-dominance criterion if

$$\left(W^{PL} - W^{MN}\right)\left(\Pi_p^{PL} - \Pi_p^{MN}\right) > \left(W^{GL} - W^{MN}\right)\left(W^{GL} - \Pi_p^{MN}\right) \tag{10}$$

Theorem 7 *The private leadership equilibrium risk-dominates the public leadership equilibrium* $\forall (c, \lambda) \in F_1$.

Then, the risk dominance criterion selects in the whole set F_1 the same equilibrium that the Pareto dominance criterion is able to select only in a subspace of F_1. It is important

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[19]In the present setting it is possible to show that the standard refinements of Nash equilibrium for normal form games – perfection, properness and strategic stability – cannot be invoked to rule out one of the pure strategy SPE.

to highlight that the risk-dominance criterion is applied to the reduced game, and not to the entire two-stage game of endogenous timing, and the two options are a priori entirely different. However, since each subgame has a unique Nash equilibrium and given the use of subgame perfection in this framework, our application of the risk-dominance criterion on the reduced game seems to us rather natural.[20] Amir and Stepanova (2006) suggest the following interpretation: the private leadership equilibrium is chosen by firms that wish to minimize the risk of coordination failure in their timing decisions.

The preference for the private leadership equilibrium is the main contribution in Beato and Mas-Colell (1984), where it is *assumed* that the public firm is committed to a decision rule (in their case the marginal-cost pricing rule), and the private firm maximizes its own profit given the decision rule of the public competitor. In the present setting, using the game with observable delay of Hamilton and Slutsky (1990) coupled with risk dominance as a selection criterion, we show that the private leadership equilibrium emerges as the *endogenous* equilibrium in the mixed duopoly $\forall (c, \lambda) \in F_2$.

4   Welfare effect of profit-maximizer public firm

In this Section we perform a comparative statics exercise in order to analyze the effects of asking the public firm to directly maximize profits[21] on welfare taking into account the result of the previous Section on the endogenous timing equilibrium.

By a change in the public firm's objective function we consider the case in which its management is instructed to maximize profits:

$$\Pi_g (q_g, q_p) = [p (q_g, q_p) - c] q_g$$

This change in the objective function might have the complementary effect of enhancing the productive efficiency of the public firm. This can happens since it is easier to measure profits than to measure welfare and with the new targets public management can be monitored and motivated in its performance. We consider the two extreme cases in which either no efficiency gain or full efficiency are achieved. In the first case, the public firm that maximize profits retains the same technology as before; in the latter, it is able to produce at the same marginal cost of its competitor, here normalized to zero. After the objective function change, the new reaction function of firm g is:

$$r_g(q_p) = \max \left\{ \frac{1}{2} (1 - c - q_p), 0 \right\} \tag{11}$$

with $c = 0$ in the case of full efficiency gains. Comparing the reaction function before (7) and after the objective function change, it is easy to see that it becomes steeper. Indeed:

$$\frac{1 + \lambda}{1 + 2\lambda} > \frac{1}{2} \qquad \forall \lambda \in \left(0, \overline{\lambda}\right)$$

and only when $\lambda \to \infty$ the slope of (7) converges to (6).

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[20]See van Damme and Hurkens (2004) and Amir and Stepanova (2006) for the application of the risk-dominance criterion on the reduced game of endogenous timing models in price game duopolies.

[21]As private and privatized firm do.

Absent efficiency gains, also the intercept. With full efficiency gains the intercept increases only when $c > \frac{1}{2(1+\lambda)}$.

The change in the reaction function is not the only effect of asking the pubic firm to maximize profits. In fact, we have to consider the (possible) change in the endogenous timing equilibrium. In order to derive the SPE of the game after the change, we can rely on the results in Amir and Grilo (1999) that apply the same endogenous timing structure to a private duopoly. The following Proposition summarizes the result.

Proposition 8 *Consider a private duopoly quantity game with strategic substitutes. When the values of the parameters are in the admissible set A, the unique SPE of the endogenous timing game is the Cournot-Nash equilibrium where both firms decide to move early.*

Proof of Proposition 8.   Under Assumptions 1 and 2 no Nash equilibrium lies on the boundary, i.e. no firm produces zero output. In this case we can apply Theorem 2.2 in Amir and Grilo (1999) that proves that both firms prefer always to be simultaneous player than Stackelberg follower. So, according to point *i*) Proposition 3, the unique SPE of the endogenous timing game is the Cournot-Nash equilibrium. ∎

The intuition for this result is clear. Since the firm's profit is strictly decreasing in the rival's output, a private leader always increases its own quantity in comparison with the Cournot-Nash quantity. By the same reason, a private follower is always strictly worse off with respect to the Cournot-Nash equilibrium. Then, sequential play is only sustainable in a mixed duopoly.

The Cournot-Nash equilibrium is the solution of the system of equations (6) and (11). Quantities and price are:[22]

$$q_g^{CN} = \frac{1-2c}{3}; \qquad q_p^{CN} = \frac{1+c}{3}$$
$$Q^{CN} = \frac{2-c}{3}; \qquad p^{CN} = \frac{1+c}{3}$$

The (domestic) total surplus and the public firm's profit are:

$$V^{CN} = \frac{8 - 8c + 11c^2}{18} - K_g - K_p; \qquad \Pi_g^{CN} = \left(\frac{1-2c}{3}\right)^2$$

Recall that in the case of achieving full efficiency $c = 0$. Of course, in order to compare welfare before and after the change in the public firm's objective function, public firm profits matter. When profits increase of 1 Euro, the measure of welfare increase of $1+\lambda$ Euros.[23] In this case, total welfare in the Cournot-Nash equilibrium is:

$$W^{CN} = \frac{1}{18} \left(2\lambda + 11c^2 + 8 - 8c\left(1 + \lambda - c\lambda\right)\right) - (1+\lambda) K_g - K_p \qquad (12)$$

---

[22]Superscript CN stands for Cournot-Nash equilibrium.

[23]If we extend our analysis to privatization, in order to compare welfare before and after privatization, the price paid to the government for buying the firm matters. In fact, since we are taking into account the shadow cost of public funds, it is not indifferent whether profits are public or private, and if the government is able to raise enough money from privatization. Given the equilibrium after privatization, the more money the government is able to raise by selling the public firm, the higher the welfare after the privatization. In the first instance, we give full bargaining power to the government; i.e., it is able to extract the whole profit from the privatized firm. In this case, total welfare in the Cournot-Nash equilibrium is the same computed considering that public firm maximizes profits, without privatization.

The following theorem states the result of the comparison when no efficiency gain occurs.

Theorem 9 *Consider a mixed duopoly game in which the order of moves is endogenous. In addition, assume that by changing the objective function of the public firm no efficiency gains are achieved. Then, maximizing profits instead of maximize welfare always reduces welfare.*

This result is in sharp contrast with those obtained *assuming* simultaneous playing. For example, de Fraja and Delbono (1989) show that assuming Cournot competition privatization[24] may enhance welfare absent efficiency gains.[25] The same result holds in the framework of the present paper. Disregarding the endogenous timing game, and comparing W^{MN} from equation (9) and W^{CN} from equation (12) $\forall (c, \lambda) \in A$, maximizin profits (or privatization) may increase welfare. More precisely, it occurs when

$$c > \tfrac{4\lambda + 6\lambda^2 + 1}{26\lambda + 12\lambda^2 + 8}$$

Now, we move the analysis to the other extreme case: full efficiency. The following Theorem formalizes the result.

Theorem 10 *Consider a mixed duopoly game in which the order of moves is endogenous. In addition, assume that by changing the objective function of the public firm full efficiency gains are achieved. Then, there exists a subset of the parameter $J \subset A$, such that asking the public firm to maximize profits reduces welfare.*

In Figure 4 we graph the set J in the parameters' space in the case of welfare is reduced by the change of the public firm's objective function even if we assume to achieve full efficiency. Endogenizing the timing of competition, before and after the change of the public firm' objective function, enlarges this space with respect to the simultaneous case. In fact, it is easy to show that, assuming simultaneous competition, welfare has been reduced if

$$c < \tfrac{3(1+2\lambda)^2 - (1+3\lambda)\sqrt{2(3+8\lambda(1+\lambda))}}{3(1+\lambda)(3+8\lambda)}$$

It is interesting to notice that the level of c such that public ownership is the dominant solution in terms of welfare is decreasing in λ. This occurs because, as λ increases, the profit motivation has a larger weight in the public firm's decision.

If we extend our analysis to privatization we can notice that the allocative efficiency effect of the public ownership decreases and the productive efficiency effect of privatization becomes more and more important. Thus, we can say that the more the public firm behaves as a profit maximizer, the better is to privatize it. This result is obtained assuming that the government is able to extract the whole profit from the new owners of the privatized firm. Suppose now that the government is able to take just half of the profit. How the previous result are affected? In Figure 4 we can see how the space of the parameters such that the privatization reduces welfare is enlarged. The dashed line delimits the space of a welfare-reducing full efficient privatization when the government sells the public firm at a price equal to half of the future profits. An extreme result occurs when the firm is sold for free. In this latter case, a full efficient privatization always lowers welfare.

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[24]Recall that under our model setting, privatization and profit maximization are equivalent in terms of market performance.

[25]This result is obtained in a different setting with symmetric firms and increasing marginal costs.
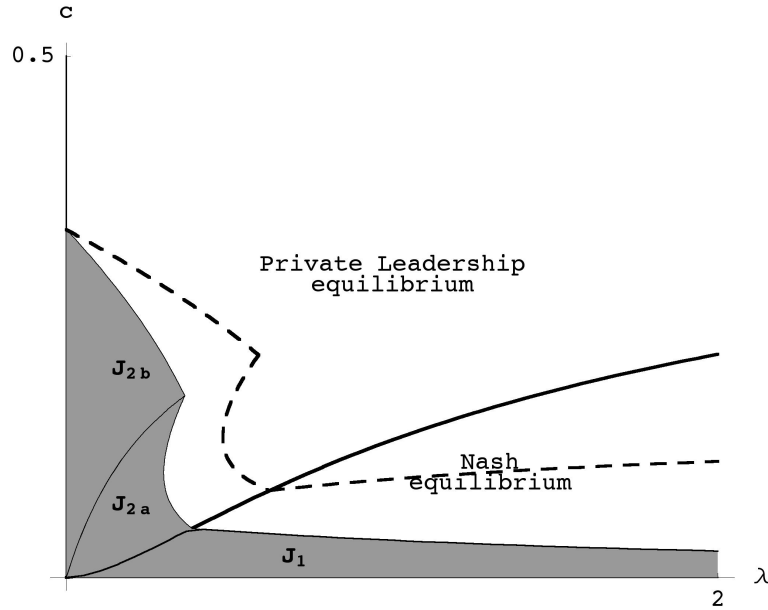
Figure 4: The welfare effect of a full efficient privatization: in gray the space of the parameters in (c, λ) in which privatization reduces welfare when the government extracts all the profits from the privatized firm

5 Conclusions

The aim of the present work is to characterize the equilibrium and analyze the effect of asking to a public firm to maximize profits in a mixed duopoly where an (inefficient) public firm competes in the quantities with a domestic private one.

We do not assume the timing of competition *a priori*. Rather, we endogenize the determination of simultaneous (Nash-Cournot) versus sequential (Stackelberg) games by applying the Hamilton and Slutsky (1990)'s model to this mixed duopoly framework.

Since we mainly refer to public utility markets open to competition, we assume that firms' production is characterized by increasing return to scale, with fixed cost and constant marginal cost. In this framework, we define the objective function of the public firm assuming that its management is instructed to maximize welfare taking into account the shadow cost of public funds. As the following citation suggests, this approach has been generally used to characterize public monopolies running a deficit.

> [M]any public enterprises are natural monopolies, i.e. firms that exhibit increasing returns to scale. Once it has been proved desirable to run such an enterprise at all, its product should be priced at marginal cost provided the resulting deficit can be financed through lump-sum taxes. If there are not lump-sum, discrepancies between consumer and producer taxes will result in inefficiencies in the rest of the economy. (...) This has been taken as an argument for requiring the public enterprise to cover, by its own means, at least part of its deficit. (Marchand et al., 1984)

We believe that extending this approach to the mixed duopoly framework is rather natural

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and fills, at least partially, some gaps of the previous literature. Indeed, discussing the results of their paper, Beato and Mas-Colell (1984, p. 82) state:

> Finally, the limitations of this paper and the need for further work should be clear. We have, for example, ruled out both fixed cost and the general equilibrium effects of distortions in other markets. We don't know if reasonable versions of the main results of this paper [...] are available in these richer settings.

The extensive process of privatization started in the eighties of the last century and still in place nowadays is essentially driven by the belief that private discipline and profit motivation can enhance efficiency. Moreover, privatization is also considered as a powerful instrument to raise money to reduce distortionary taxation. In this work we contrast the general extent of these ideas. We show that, absent efficiency gains, privatization never increases welfare, and that an inefficient public firm may be preferred even when large efficiency gains could be realized by privatization. These results are obtained assuming that both public firm's profits and privatization proceeds are substitute for distortionary taxation. The endogenous timing model applied to the mixed oligopoly framework is not less important for our results. While after privatization only the simultaneous (Nash-Cournot) equilibrium can be implemented, with a public firm sequential equilibria – that are always welfare superior – may be sustained as SPE of the endogenous timing game. Therefore, privatization changes not only the ownership and the objective function of the public firm, but also the type of competition in the market.

Finally, the assumption of larger marginal cost for the public firm deserves a last comment. In our model we follow the general presumption that public ownership is relatively inefficient when compared to private ownership. This has been justified by the theory of incentives that has been used to demonstrate that agency problems in state-owned enterprises can cause larger inefficiencies than in private-owned firms. But we are aware that from an empirical point of view the picture is quite mixed and the variance of the results substantial (Cuervo and Villalonga, 2000).[26] However, any relaxation of our assumption obviously strengthen the results obtained in this paper.

6   Appendix

Proof of Lemma 4.   Comparing the equilibrium profits Π_p^{GL} in Table 2 with Π_p^{MN} in equation (8), it easy to check that $\forall \lambda \geq 0$:

(i) $\forall c \in \left(0, \frac{1+2\lambda}{4(1+\lambda)}\right)$

$$\Pi_p^{GL} - \Pi_p^{MN} > 0 \qquad \forall c > \bar{c}(\lambda)$$

where

$$\bar{c}(\lambda) = \frac{\lambda^2}{2\lambda^2 + 3\lambda + 1} \qquad \text{with } \frac{\partial \bar{c}(\lambda)}{\partial \lambda} > 0.$$

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[26]See for example the reviews of Megginson and Netter (2001) and Willner (2001) that report the results of hundreds of empirical papers on privatization and on the comparison of private and public ownership, and Newbery (2000, chapter 3) that summarizes empirical findings on the technical and economic efficiency of private and public firms in utility markets.

(ii) $\forall c \in \left[\frac{1+2\lambda}{4(1+\lambda)}, \frac{1}{2}\right)$, $\Pi_p^{GL} - \Pi_p^{MN} > 0$ always.

Thus, we define the subspace F_1 and $\widehat{F}_1$ as follows:

$$F_1 = \{(c,\lambda) \subseteq A | c > \overline{c}(\lambda)\} \quad \text{and} \quad \widehat{F}_1 = \{(c,\lambda) \subseteq A | c \leq \overline{c}(\lambda)\} \tag{13}$$

■

Proof of Lemma 5. Comparing the welfare level W^{PL} in Table 3 with W^{MN} in equation (9), it easy to check that $\forall \lambda \in (\overline{\lambda})$:[27]

(i) $\forall c \in \left(0, \frac{\lambda}{3\lambda+1}\right)$

$$W^{PL} - W^{MN} > 0 \qquad \forall c > \underline{c}(\lambda)$$

where

$$\underline{c}(\lambda) = \frac{3\lambda^2 + 7\lambda^3}{21\lambda + 34\lambda^2 + 17\lambda^3 + 4} \qquad \text{with } \frac{\partial \underline{c}(\lambda)}{\partial \lambda} > 0$$

(ii) $\forall c \in \left[\frac{\lambda}{3\lambda+1}, \frac{1}{2}\right)$,

$$W^{PL} - W^{MN} > 0 \qquad \forall \lambda \in (0, \overline{\lambda})$$

Thus, we define the subspace F_2 and $\widehat{F}_2$ as follows:

$$F_2 = \{(c,\lambda) \subseteq A | c > \underline{c}(\lambda)\} \quad \text{and} \quad \widehat{F}_2 = \{(c,\lambda) \subseteq A | c \leq \underline{c}(\lambda)\} \tag{14}$$

■

Proof of Theorem 6.

a) When $(c,\lambda) \in \widehat{F}_1$ the private firm prefers the mixed duopoly Nash equilibrium to the public leadership equilibrium. When $(c,\lambda) \in \widehat{F}_2$ the public firm is better off in the Nash equilibrium than in the private leadership equilibrium. Therefore, in the intersection space $\widehat{F}_1 \cap \widehat{F}_2$, no firm wants to be follower. Since $\forall \lambda \in (0, \overline{\lambda})$, $\underline{c}(\lambda) < \overline{c}(\lambda)$, it follows that $\widehat{F}_2 \subset \widehat{F}_1$; then $\widehat{F}_1 \cap \widehat{F}_2$ coincides with $\widehat{F}_2$. Given that each player always prefers to be the Stackelberg leader than a simultaneous player, point i) of Proposition 3 applies and the mixed duopoly Nash equilibrium is the unique SPE of the endogenous timing game.

b) When $(c,\lambda) \in \widehat{F}_1$ the private firm prefers the mixed duopoly Nash equilibrium to the public leadership equilibrium. When $(c,\lambda) \in F_2$ the public firm is better off in the private leadership equilibrium than in the mixed duopoly Nash equilibrium. So, point iii) of Proposition 3 applies and the Stackelberg equilibrium with the private firm acting as leader is the unique SPE of the endogenous timing game.

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[27]The threshold $\underline{c} < \frac{\lambda}{3\lambda+1}$ $\forall \lambda < 5.37228$. Since λ is a measure of the distortion by taxation, we are comfortable assuming that $\overline{\lambda}$ is lower than 5.37228. If $\lambda \geq 5.37228$ we would have that $W^{PL} < W^{MN}$ always.

c) When $(c, \lambda) \in F_1$ the private firm prefers to play as Stackelberg follower than to play simultaneously. When $(c, \lambda) \in F_2$ the public firm prefers to play as Stackelberg follower than to play simultaneously. Since $\forall \lambda > 0$, $\underline{c}(\lambda) < \overline{c}(\lambda)$, it follows that $F_1 \subset F_2$; then $F_1 \cap F_2$ coincides with F_1. So, point $ii)$ of Proposition 3 applies and both Stackelberg equilibria belong to the set of the (pure strategy) SPE of the endogenous timing game.

∎

Proof of Theorem 7. In order to prove the result we need to consider three cases depending on the fact that boundary solutions may occur in the two sequential equilibria. By comparing the thresholds defined in Section 3.2, we have the following equilibria:

(i) when $(c, \lambda) \in F_1$ and $c < \frac{\lambda}{1+3\lambda}$, both Stackelberg equilibria are interior. Then, the values of W^{GL}, Π_p^{GL}, W^{PL} and Π_p^{PL} of interest are those in the first row of Tables 2 and 3.

(ii) when $(c, \lambda) \in F_1$ and $\frac{\lambda}{1+3\lambda} < c < \frac{1+2\lambda}{4(1+\lambda)}$, the public firm does not produce in the private leadership equilibrium while it produces positive quantity in the public leadership equilibrium. Then, the values of W^{GL} and Π_p^{GL} of interest are are those in the first row of Table 2, while for W^{PL} and Π_p^{PL} we have to consider the values in the second row of Table 3.

(iii) when $(c, \lambda) \in F_1$ and $c > \frac{1+2\lambda}{4(1+\lambda)}$, the public firm does not produce in both Stackelberg equilibria. Then, the values of W^{GL}, Π_p^{GL}, W^{PL} and Π_p^{PL} of interest are those in the second row of Tables 2 and 3.

Applying the criterion (10), straightforward but tedious computations show the result. ∎

Proof of Theorem 9. In order to prove the result, we need to consider three cases: (i) Nash is the relevant equilibrium of the mixed duopoly; (ii) private leadership is the relevant equilibrium of the mixed duopoly with an interior solution; and (iii) private leadership with the public firm not producing is the relevant equilibrium of the mixed duopoly.

(i) By point a) in Theorem 6 Nash is the relevant equilibrium of the mixed duopoly game when $(c, \lambda) \in \widehat{F}_2$. So, we have to compare W^{MN}, defined in equation (9) with W^{CN} (equation 12). Straightforward computations show that

$$W^{MN} > W^{CN} \qquad \forall (c, \lambda) \in \widehat{F}_2$$

(ii) By points b) and c) in Theorem 6 and Theorem 7 the Stackelberg outcome with the private firm as leader is the relevant SPE of the mixed duopoly game when $(c, \lambda) \in F_2$. Moreover, when $c < \frac{\lambda}{3\lambda+1}$ the public firm produces positive quantity in the equilibrium. Then, we have to compare the value of W^{PL} in the first row of Table 3 with W^{CN} (equation 12). Straightforward computations show that

$$W^{PL} > W^{CN} \qquad \forall (c, \lambda) \in F_2, \quad c < \frac{\lambda}{3\lambda + 1}$$

20

(iii) When $c \geq \frac{\lambda}{3\lambda+1}$, the public firm does not produce in the private leadership equilibrium. Thus, we have to compare the value of W^{PL} in the second row of Table 3 with W^{CN} (equation 12). Straightforward computations show that

$$W^{PL} > W^{CN} \qquad \forall (c, \lambda) \in F_2, \quad c > \frac{\lambda}{3\lambda + 1}$$

∎

Proof of Theorem 10. When the privatized firm achieves full efficiency gains, welfare after privatization is:

$$W^{CN}\big|_{c=0} = \frac{4 + \lambda}{9} - (1 + \lambda) K_g - K_p \tag{15}$$

In order to prove the result we have to distinguish between three cases as in Theorem 9.

(i) By point a) in Theorem 6 Nash is the relevant equilibrium of the mixed duopoly game when $(c, \lambda) \in \widehat{F}_2$. So, we have to compare W^{MN}, defined in equation (9) with W^{CN} (equation 15). Straightforward computations show that

$$W^{MN} \geq W^{CN}\big|_{c=0} \qquad \forall (c, \lambda) \in \widehat{F}_2, \quad c < \frac{3(1+2\lambda)^2 - (1+3\lambda)\sqrt{2(3+8\lambda(1+\lambda))}}{3(1+\lambda)(3+8\lambda)}$$

Thus, we can define the subset

$$J_1 = \left\{ (c, \lambda) \in \widehat{F}_2 \,\middle|\, c < \frac{3(1+2\lambda)^2 - (1+3\lambda)\sqrt{2(3+8\lambda(1+\lambda))}}{3(1+\lambda)(3+8\lambda)} \right\}$$

Referring to the definition of the subset $\widehat{F}_2$ in (14), it is easy to check that J^1 is a nonempty set.

(ii) By points b) and c) in Theorem 6 and Theorem 7 the private leadership equilibrium is the relevant SPE of the mixed duopoly game when $(c, \lambda) \in F_2$. Moreover, when $c < \frac{\lambda}{3\lambda+1}$ the public firm produces positive quantity in the equilibrium. Then, we have to compare the value of W^{PL} in the first row of Table 3 with W^{CN} (equation 15). First of all, define

$$F_{2a} = \left\{ (c, \lambda) \in F_2 \,\middle|\, c < \frac{\lambda}{3\lambda + 1} \right\}$$

Straightforward computations show that:

$$\begin{aligned} W^{PL} &\geq W^{CN}\big|_{c=0} \\ \forall (c, \lambda) &\in F_{2a}, \quad 9c^2 (1 + \lambda)^2 (4 + 9\lambda) - 18c\lambda (1 + \lambda) (2 + 3\lambda) + 4\lambda - 7\lambda^3 > 0 \end{aligned}$$

Thus we can define the subset

$$J_{2a} = \left\{ (c, \lambda) \in F_{2a} \,\middle|\, 9c^2 (1 + \lambda)^2 (4 + 9\lambda) - 18c\lambda (1 + \lambda) (2 + 3\lambda) + 4\lambda - 7\lambda^3 > 0 \right\}$$

that is nonempty.

21

(iii) Defining

$$F_{2b} = \left\{ (c, \lambda) \in F_2 | c \geq \frac{\lambda}{3\lambda + 1} \right\}$$

the public firm does not produce in the private leadership equilibrium. Then, we have to compare the value of W^{PL} in the second row of Table 3 with W^{CN} (equation 15). Straightforward computations show that the subset $J^{2b} \subset F_{2b}$ such that privatization reduces welfare is not empty.

$$J_{2b} = \left\{ (c, \lambda) \in F_{2b} | c < \frac{1}{3} \sqrt{1 - 2\lambda} \Leftrightarrow W^{\Pr L} - W^{FE} \geq 0 \right\}$$

Then, the subset of parameters' values such that a full efficient privatization with full bargaining power to the government reduces welfare is the following:

$$J = J_1 \cup J_{2a} \cup J_{2b}$$

∎

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23

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