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**CAN MIGRANTS SAVE GREECE FROM AGEING? A
COMPUTABLE GENERAL EQUILIBRIUM APPROACH
USING G-AMOS.**

BY

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Can Migrants save Greece from Ageing? A Computable General Equilibrium Approach using G-AMOS.

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Abstract: The population of Greece is projected to age in the course of the next three decades. This paper combines demographic projections with a multi-period economic Computable General Equilibrium (CGE) modelling framework to assess the macroeconomic impact of these future demographic trends. The simulation strategy adopted in Lisenkova et. al. (2008) is also employed here. The size and age composition of the population in the future depends on current and future values of demographic parameters such as the fertility, mortality rates and the level of annual net migration. We use FIV-FIV software in order to project population changes for 30 years. Total population and working age population changes are introduced to the G-AMOS modelling framework calibrated for the Greek economy for the year 2004. Positive net migration is able to cancel the negative impacts of an ageing population that would otherwise occur as a result of the shrinking of the labour force. The policy implication is that a viable, long-lasting migration policy should be implemented, while the importance of policies that could increase fertility should also be considered.

JEL classification: J11, J21

Key words: CGE modelling, ageing population, migration, demography, Greece

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1. Introduction

As in most western countries, population in Greece is projected to decline and age. Greece has traditionally been a country of out-migration. However, in the last two decades the population outflow has reversed. This has generated a stock of foreign population, which is estimated to be close to 10% of the nationals¹ (Kontis et al., 2006). This has been a real demographic shock. In contrast, the country has experienced a declining fertility rate (1.3 in 2004) and a continuously increasing life expectancy. Fertility rates in Greece have been lower than the European average until the end of the 60's and peaked during the 70's but never achieved the levels reached in the baby boom period in Europe. The baby boom in Greece was late and small due to the political instability of the post World War II period. However, the fertility rate has largely converged to the European average during the last two decades.

If the assumptions of ESYE (National Statistics Agency) on future trends of demographic variables (fertility, mortality, migration) materialise, then in the course of the next three decades (by 2034) is projected to increase slightly by 0.54%. However at the same time working age population is expected to decrease by 6.13%. Under the same assumptions but without the presence of migration total population will fall by 11.5% and working age population by 19.76%. We are going to try and analyse the impacts, of these future demographic trends. At present very little attention is paid to the macroeconomic impacts that these will have in the foreseeable future.

Existing literature focuses on the impacts of immigration upon the economy. Lianos (1996) found that wages of (illegal) immigrants in Northern Greece were 40-60% lower than the ones of the local population. He also found that the vast majority of immigrants take jobs that the locals would never do. This study is rather descriptive, and thus does not assess the impact of immigration on macroeconomic variables in a quantifiable way. Sarris and Zografakis (1999) use a Computable General Equilibrium model to assess the impact of illegal immigration upon the Greek economy. In their exercise, population stock is only adjusted by a one shot influx of illegal immigrants in the Greek economy. They find that almost a third of the

¹ Contrary to many countries Greece defines foreigners "by blood" and not "by birth". So, there are people that have been born in the country but have not got citizenship. In contrast, others have not been born and brought up in the country but are granted citizenship. So, when we refer to "foreigners" we may include people that have been born in the country.

population is adversely affected from illegal immigration. However, the negative impact disappears when they model labour market rigidities. (Fixed nominal wage). The paper at hand adds to the relevant literature in linking demographic projections with consistent economic modelling. Thus it sheds light on the effect of current and future fertility, mortality and migration trends with the use of consistent economic and demographic modelling.

While demographic changes have many economic implications that are related to public expenditure, expected increase in demand for health care, funding of pensions of a growing aged population etc., the focus of this paper is to assess the impact of future demographic trends upon the economy through the consequent changes in the labour market. We combine a demographic model, (FIV-FIV) and a Computable General Equilibrium (CGE) model of the Greek Economy (G-AMOS) calibrated for the year 2004. The projection and simulation horizon is three decades, a span that is long enough to allow for the population changes to be fed into the rest of the economy and at the same time not too long to make our population projections completely unreliable.

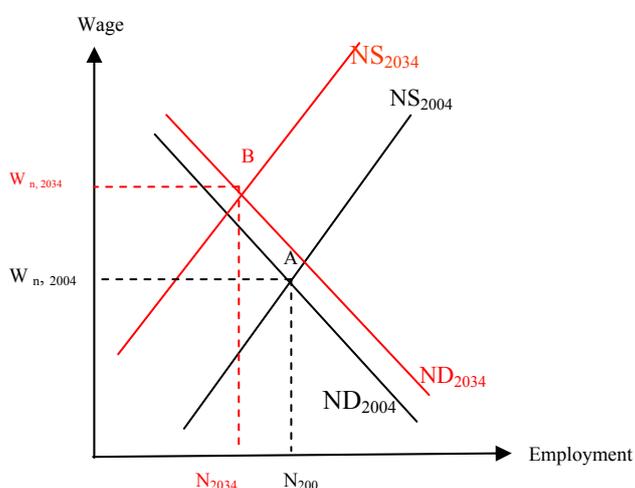
The remainder of the paper is structured as follows. The next section provides the theoretical framework upon which our analysis is based. The two following sections provide details on the demographic and the economic model. The economic impact of the base projection is discussed in section 5. Section 6 focuses on sensitivity analysis that we have performed by varying values of demographic as well as economic parameters. Section 7 concludes.

2. Theoretical setting

In this section we outline the theoretical foundation for the simulation results that are reported in the subsequent sections. The focal point is the labour market, which is analysed using an aggregate labour demand and supply framework (Lisenkova et.al., 2008). We present a long run approach, in order to identify the direction in which we expect the equilibrium wages and employment to move as the time passes. Figure 1 represents the interaction of the labour supply and general equilibrium labour demand curves in the unified Greek labour market, a unified labour market, in which workers can freely move between the identified sectors. When the working age population is unchanged (in our case this is equivalent to an unchanged labour force), the quantity

of labour supplied can only increase if the real wage is increased². The general equilibrium Labour demand is sloping downwards and this is the case for most of the research that has utilised the AMOS modelling framework (Gillmartin et. al. 2007, Lisenkova et. al. 2008, Turner, 2002). However, for combinations of extreme product demand and factor substitution elasticities, the general equilibrium labour demand curve can be upward sloping (McGregor *et al*, 1995). Note that the endogenously determined level of household income will also affect labour demand in the general equilibrium.

Figure 1- Impact of an ageing and declining population



In Figure 1 we compare the long-run labour market equilibrium in 2034 with that in 2004, under the assumption that the base population projection applies and that real *per capita* expenditure of the Greek government remains constant. As mentioned before in the simulation exercise there is no demand shock imposed associated with the ageing of the population. Instead, the assumption is that government expenditure per capita remains constant. One would expect that consumer and government demand patterns are age specific and therefore shifting demographics would alter the industrial composition of consumer and government demand. However, for now we will assume that government expenditure per head remains constant over the simulation period. That is government expenditure follows the changes of total population. The initial equilibrium is represented by point A, where the base-period

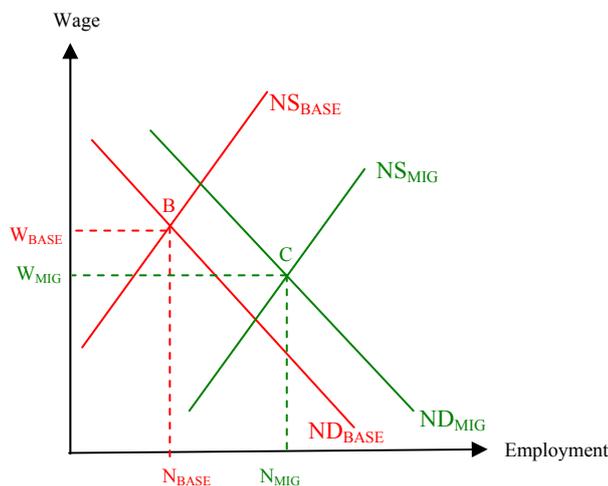
² We actually use a wage curve specification for the labour market. (Blanchflower and Oswald, 1994; Layard *et al*. 1991).

(2004) labour demand and supply curves ND_{2004} and NS_{2004} intersect. This generates the initial equilibrium employment and real wage levels $w_{n,2004}$, N_{2004} .

As mentioned before, over the period up to 2034, the total population in Greece will increase marginally but at the same time age. First, the lower working age population reduces labour supply at each wage level generating an inward shift of the labour supply curve. The new labour supply curve NS_{2034} , is on the left of the original labour supply curve NS_{2004} . On the other hand total population changes will affect labour demand through changes in government expenditure. The new demand curve is on the right of the original because during this period government expenditure (an element of final demand) increases in line with the slight increase of total population.

The new equilibrium B, is at the intersection of the general equilibrium labour demand and supply curves ND_{2034} and NS_{2034} . Both labour supply and labour demand shift have pushed the real wage upwards. However, while increased labour demand tends to increase employment while decreased labour supply tends to decrease it. We expect the supply effect to be of greater magnitude in this case.

Figure 2 - The Labour Market in 2034 comparing base projections and higher positive net migration every year



In Figure 2 we expand our investigation to show the impact on the labour market in 2034 of increased inward migration. Figure 2 compares the labour market in 2034 under two different scenarios. One simply takes the base population projection. The second represents a situation in which there is higher positive net in-migration. This is

a policy parameter that can be altered immediately at the will of the Greek government³.

The situation under the base projection is given by point B, which is the intersection of the labour demand and supply functions NS_{2034} , ND_{2034} . This was the final position illustrated in Figure 1. Now impose net in-migration at a rate higher than that assumed under base scenario. Working age population, and therefore the labour force, is going to be higher than under the base prediction for 2034. This is illustrated in Figure 2 by an outward shift of the labour supply curve to NS_{MIG} . Labour demand will also shift outwards to ND_{MIG} as a result of higher population-linked government expenditure. The new equilibrium is at point C, the intersection of the new labour supply and demand curves. In this case the shifts in labour demand and supply both work to increase equilibrium employment (compared to the equilibrium at B). Further, because we expect that the increase in labour supply will dominate the increase in labour demand, pressure in the labour market should ease and the wage fall.

The two figures exhibit the impact of population ageing and in-migration on the labour market. We expect these changes to have major impacts through a tightening of the labour market, and consequences upon competitiveness. These labour market changes will have major impacts on the economy overall. In particular, with an ageing population, we expect a negative effect upon competitiveness due to the increased wage, which is a major cost of production. This reduced competitiveness has a negative impact on GDP. In migration is expected to counteract the tightening of the labour market (Sarris and Zografakis, 1999). The Computable General Equilibrium framework we use allows us to explore such impacts in greater detail by consistently modelling all the complex interactions within the economy while it give us a sufficient degree of detail in terms of results.

³ An extensive discussion of the current migration policy in Greece can be found in Kontis et. al. (2006) as well as in Katrougalos et.al. (2004)

3. The FIV-FIV software

The software produces population projection by using the cohort component method, which is the most widely used. The cohort component method tracks each cohort and its mortality, fertility, and migration over time. Starting with a base population, year-by-year deaths are subtracted, and births and net migration are added to the population. This program allows us to project future population by sex and age structure, based on the current age-sex structure and additional assumptions about main demographic variables: mortality, fertility and migration. Projections are made in five-year cycles and can involve five to infinite years. Births create a cohort of the age 0-4 at the end of the cycle. The rest of the cohorts face a probability of mortality and their survivors are five years older in the end of the cycle. Net migration is algebraically added to cohorts during the cycle. The defined cohorts are presented in the following table.

Table 1 – Cohorts identified in the FIV-FIV software

Cohort index	Initial Age	Final Age	Cohort index	Initial Age	Final Age
0	Born	0-4	9	40-44	45-49
1	0-4	5-9	10	45-49	50-54
2	5-9	10-14	11	50-54	55-59
3	10-14	14-19	12	55-59	60-64
4	14-19	20-24	13	60-64	65-69
5	20-24	25-29	14	65-69	70-74
6	29-30	30-34	15	70-74	75-79
7	35-39	40-44	16	75+	80+
8	40-44	45-49			

In the end of every cycle, the population of the cohorts 15 and 16 are added together to form the group 75+. Projections are separately done for men and women. The first step is to calculate the population alive in the beginning of the cycle, which comprises of cohorts one to sixteen. The second step is to calculate the population of cohort zero which is the remaining and is born during the cycle. Population alive is calculated by taking into account the effect of mortality for each cohort. That is:

$$(1) C_{s,c}^F = C_{s,c}^I * S_{s,c}$$

Where, C stands for the population of cohort c, sex s at a date 0 or five indicated by the superscript. S is the survival rate for each cohort. Migration is incorporated the following way. It is assumed that migrants enter or leave the country following a uniform distribution during the five year cycle. So, migrants are exposed to mortality for an average of one half of the cycle. Therefore for migration to be taken into account the equation above should be augmented the following way.

$$(2) C_{s,c}^F = C_{s,c}^I * S_{s,c} + M_{s,c} * (1 + S_{s,c})/2 \quad \text{for the cohorts one to sixteen.}$$

The number of births will depend on the number and age of women in childbearing age, and their age specific fertility rates. Women of the age group 15-49 are the ones considered of being childbearing age. These are seven cohorts, namely 15-19, 20-24, 25-29, 30-34, 35-39, 40-44, and 45-49. The mean number of women with a probability of having a child for each of these cohorts will be:

$$(3) P_i = (P_{F,C}^F + P_{F,C}^I)/2 \quad \text{where } i \text{ is the cohort index.}$$

The number of women is either given when we have superscript of zero, (since it is given by the previous step) or calculated survival in the preceding when superscript is five. The number of births is calculated by using age specific fertility rates, F_i , and for the whole cycle total number of births will be:

$$(4) B = 5 * \sum_4^{10} (P_{F,C} * F_C)$$

Births are calculated for males and females separately. Adjusting for the proportion of each gender and the corresponding survival rates males and females of the birth cohort are:

$$(5a) C_{f,0}^F = g * B * S_{f,0} \quad (5b) C_{m,0}^5 = (1 - g) * B * S_{m,0}$$

Where g is the proportion of females born. Again in order to augment the above equations for migration we need to make assumptions about the time and age distribution of migrants. The assumption is that migrants come in following a uniform distribution, during the five year period and that births are equally distributed between all ages 0 to 5.

This is a sufficient approximation if we assume that the probability of dying decline in a linear manner from birth until the age of 5. This is not particularly true especially for the newborns. In addition, very few migrants are of the age of some

weeks or months. The software does not require making assumptions related to infant mortality of migrants. Instead it adjusts arbitrarily the above approximation and assumes that the survival rate comes in with a factor of one third instead of a half. Equations 5a and 5b take the form:

$$(6a) C_{f,0}^5 = g * B_s * S_{f,0} + M_{f,0} * (2/3 + 1/3 * S_{f,0})$$

$$(6b) C_{m,0}^5 = (1 - g) * B_s * S_{m,0} + M_{m,0} * (2/3 + 1/3 * S_{m,0})$$

We have now calculated the new cohort and the projection to the next five-year cycle is complete. In the next step each cohort index is increased by one and the final population of cohort i is used as the initial population of cohort $i+1$.

The data we have used for the projection is published by ESYE. However there have been some gaps in data availability. For example, a problem we have faced is the fact that there is no data for the age and sex composition of migrants. We have therefore assumed that migrants have the age and sex composition of the foreign population of the country. The lack of data, that has been mentioned elsewhere (Kontis et al., 2006) as well as the software structure has caused small discrepancies between the ESYE Base projection and the projection we have generated under the same assumptions for the demographic variables. We are going to refer to this projection as the “Base scenario” although there are small discrepancies. The size of these deviations is negligible in relation to the question at hand.

4. G-AMOS: The Economic Model

G-AMOS stands for Greek-AMOS, which uses the AMOS modelling framework. (*A Macro-Micro Model Of Scotland*) and has been developed by a team of researchers in the University of Strathclyde. It is referred to as a Computable General Equilibrium modelling framework “... because it encompasses a range of behavioural assumptions, reflected in equations which can be activated and configured in many different ways” (Harrigan *et al*, 1991, p. 424). A wide range of model closures as well as parameter values is available to the user, as appropriate for particular applications. A good general description of CGE modelling is given in Greenaway *et al* (1992) and an extensive review of regional CGE models can be found in Partridge and Rickman (1998).

We have calibrated the G-AMOS modelling framework to data on the Greek economy given in the form of a Social Accounting Matrix (SAM) for the year 2004. The model has 3 transactor groups - households, firms and government - and 2 exogenous external transactors – the rest of the EU (REU) and the rest of the world (ROW). The model has 25 activities/commodities and these are listed in Table A1.1 in Appendix 1. A condensed account of the model structure is given in Table A2.1.

In the version of AMOS used here, production takes place in perfectly competitive industries using multi-level production functions. This means that in every time period all commodity markets are in equilibrium, with price equal to the marginal cost of production. Value-added is produced using capital and labour via standard production function formulations so that, in general, factor substitution occurs in response to relative factor-price changes. Typically, constant elasticity of substitution (CES) technology is adopted, which is the case in simulations reported here.⁴ In each industry intermediate purchases are modelled as the demand for a composite commodity with fixed (Leontief) coefficients. These are substitutable for imported commodities via an Armington link. The composite input then combines with value-added (capital and labour) in the production of each sector’s gross output. Cost minimisation drives the industry cost functions (equation 1 in Table A2.1) and the factor demand functions (equations 7 and 8 in Table A2.1).

⁴ Leontief and Cobb-Douglas options are available as special cases.

Whilst the AMOS framework offers a wide choice of labour market closures, in the simulations reported here the labour market is characterised by a regional bargaining function (also expressed as a wage curve, represented as equation 5 in Table A2.1). This establishes a negative relationship between the real wage and the unemployment rate (Minford *et al.*, 1994). Empirical support for this wage curve specification is now widespread (Blanchflower and Oswald 1994). The bargaining function is parameterised using the regional econometric work reported in Layard *et al.* (1991):

$$\ln rw_{n,t} = a - 0.113 \ln u_t + 0.40 \ln rw_{n,t-1}$$

where rw is the Greek real wage, u is the Greek unemployment rate, t is the time subscript and a is a calibrated parameter.⁵ To transform the real wage to the nominal wage, we multiply by the consumer price index (equation 2 in Table A2.1). Nevertheless, sensitivity analysis has been performed in section 6 where we examine extreme cases for the closure of the labour market.

Perfect labour mobility is assumed between sectors, generating a unified labour market. Therefore, although wage rates vary between sectors in the base-year data set, in the simulations wages in all sectors change by the same proportionate amount in response to exogenous shocks. The nominal wage in each time period is then derived through the interaction of the resulting wage curve and the general equilibrium labour demand curve (equation 9 in Table A2.1). In the derivation of the general equilibrium labour demand curve, it is important to note that all prices and incomes are taken to be endogenous.

The four main components of commodity final demand (represented by equation 12 in Table A2.1) are consumption, investment, government expenditure and exports. Household consumption is a linear homogenous function of real disposable income and relative prices (equations 2, 11 and 13 in Table A2.1). Real government expenditure per head is assumed to be constant (equation 17, Table A2.1) and in these simulations the population is determined exogenously using the demographic model.

⁵ The calibration is made so that the model, together with the set of exogenous variables, will recreate the base year data set. This calibrated parameter does not influence simulation outputs, but the assumption of initial equilibrium is, of course, important.

Exports are determined by exogenous external demand via an Armington link, making exports relative price sensitive (equation 18, Table A2.1).

The modelling of investment demand is a little more complex. In the multi-period variant of the model, capital stock adjustment at the sectoral level, which ultimately determines aggregate investment demand, is dealt with in the following way. Within each time period, both the total capital stock and its sectoral composition are fixed. The interaction between this fixed capital supply and capital demand at the sectoral level determines each sector's capital rental rate (equation 10, Table A2.1). The capital stock in each sector is then updated between periods via a simple capital stock adjustment procedure, according to which investment equals depreciation plus some fraction of the gap between the desired and actual level of the capital stock (equations 6, 14 and 15 in Table A2.1).⁶ Desired capital stocks are determined on cost-minimisation criteria, using the user cost of capital as the relevant price of capital (equations 3 and 4 in Table A2.1). In the base period the economy is assumed to be in long-run equilibrium, where desired and actual capital stocks are equal, with investment simply equal to depreciation. Investment as a source of product demand is then determined by running the demand for increased capital stock by sector through the capital matrix (equation 16, Table A2.1).

We interpret the conceptual time periods of the model as years: annual data are used for the calibration and, where applicable, the estimation of parameter values.

As stated earlier in this section, the structural characteristics of the AMOS model are parameterised on a Social Accounting Matrix (SAM) for Greece for 2004. In all sectors, the elasticity of substitution between capital and labour in the production of value added is 0.3 although sensitivity analysis is conducted. Intermediate composite goods are assumed to be produced by a Leontief type production procedure with fixed coefficients. This is required because of the large number of zero entries. The default Armington trade elasticities for imports and exports are 8.0. This is to reflect the fact that the country is considered to be a "price taker" in international markets. Sarris and Zografakis (1999) also applied a high value of Armington elasticity while Ioakimoglou (1999) econometrically estimated that 80% of variation of prices in Greece is attributable to international price changes. Nevertheless sensitivity analysis

⁶ This process of capital accumulation is compatible with a simple theory of optimal firm behaviour given the assumption of quadratic adjustment costs. The whole process is analogous to Tobin's q .

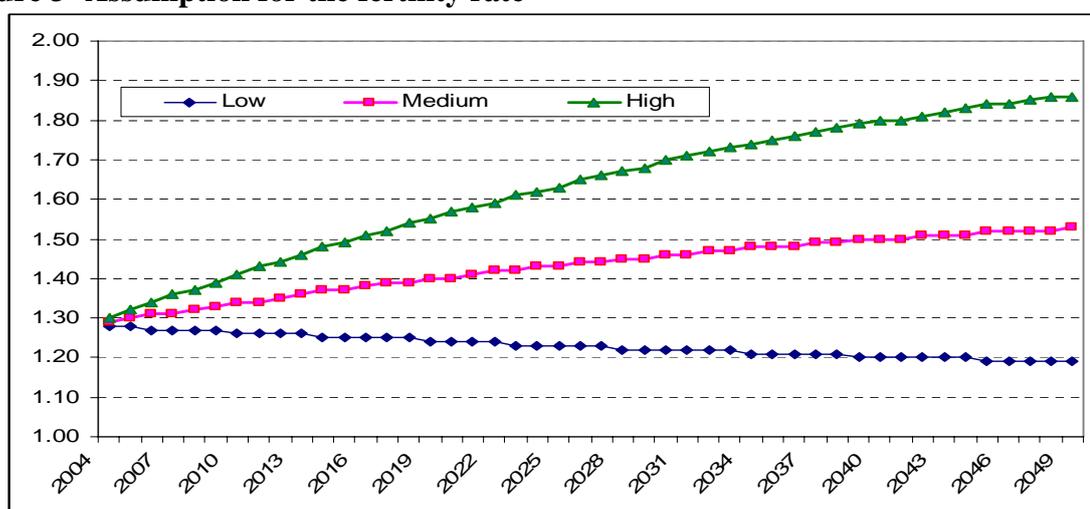
using the values of 2, 5 and 8 is performed later on. The speed of adjustment parameter for the adjustment of actual to desired capital stock is 0.5.

Before discussing the simulation results it is important to clarify a key characteristic of the G-AMOS model. G-AMOS is not a forecasting model. When it is parameterised on the base year data set, it is assumed that the economy is in long-run equilibrium. If there are no changes to the exogenous variables and the model is run in period-by-period mode, then the model will simply report an unchanging economy. Results are presented as percentage changes from the base year value and thus demonstrate deviations from what would occur if “nothing happened” in the economy.

5. The Consequences for the Greek Economy of the Base scenario for the projected population

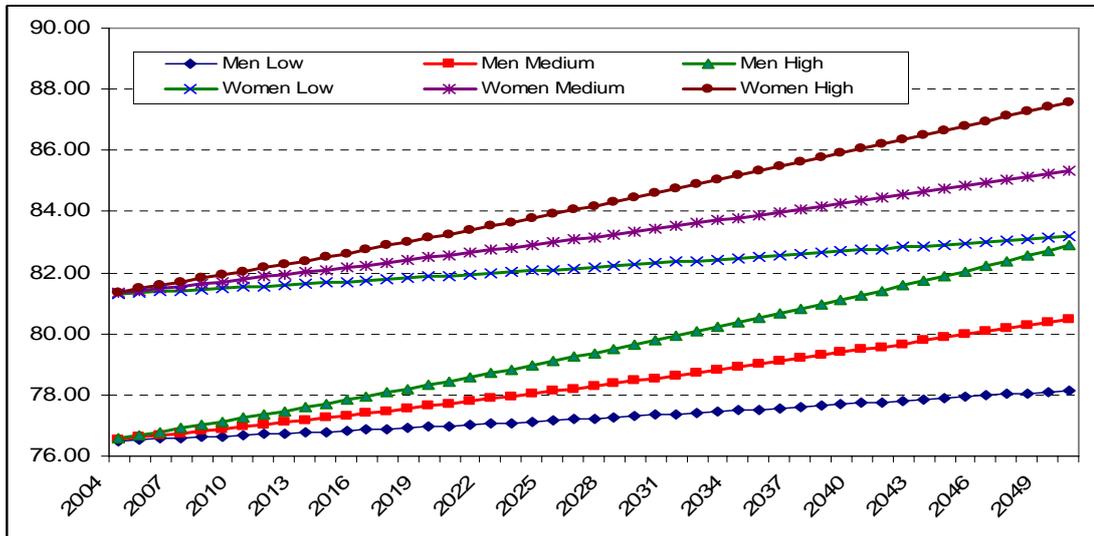
ESYE has examined three scenarios of future demographic trends. These are labelled low, medium, high and the related assumptions on the values of demographic parameters are shown in figures 3,4 and 5. Fertility is recorded to be 1.29 in the year 2004. Under the low scenario it assumed to fall to 1.19 children per woman in childbearing age by 2050. For the medium and high scenarios the corresponding figures are 1.53 and 1.87 respectively. Figure 3 below shows the assumed trends.

Figure 3- Assumption for the fertility rate



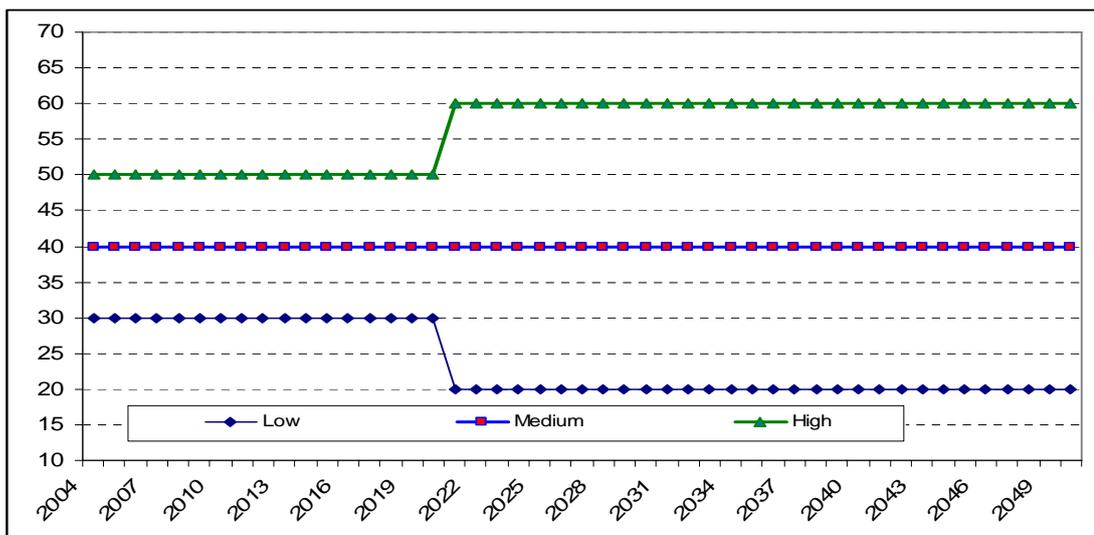
Life expectancy is assumed to rise in a linear manner and to reach for men 78.15 years, 80.49 years, 82.89 years in 2050 for the low medium and high scenarios respectively. The corresponding numbers for women are 83.19, 85.32, 87.57. Below we see the graphical representation of the assumptions being made.

Figure 4- Life expectancy assumptions



Finally, the medium scenario assumes that net-migration will remain constant at the current level of 40,000 per annum. In the high scenario the assumption is

Figure 5- Assumptions for net migration



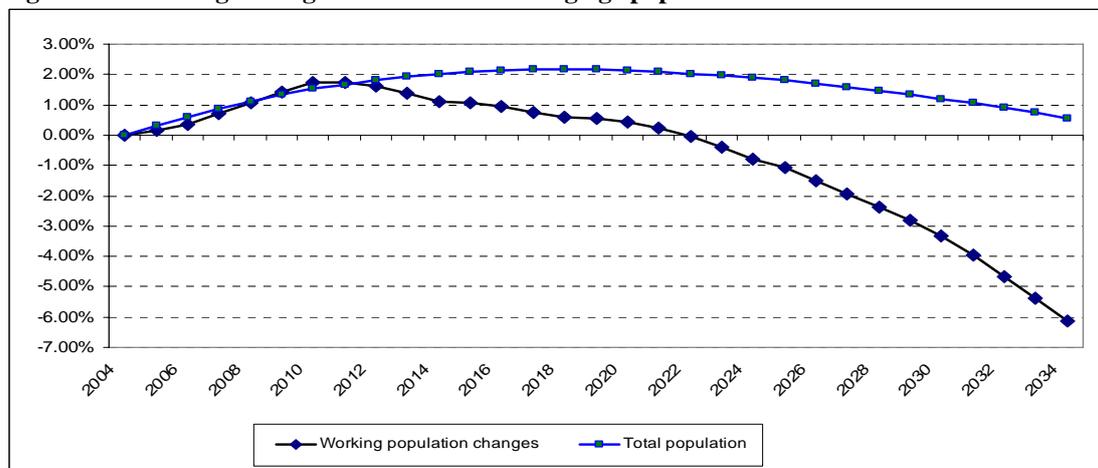
Using the medium values for fertility, life expectancy and net migration we have generated a population projection for three decades after our base year which sufficiently approximates the projected population by ESYE under the medium

assumptions. The table below shows changes for total and working age population for five year periods and the figure shows the corresponding trajectories for our “base scenario”.

Table 2- Total and working age population under the base scenario

	2004	2009	2014	2019	2024	2029	2034
16-64	7344	7448	7425	7384	7287	7139	6894
% change	0.0%	1.4%	1.1%	0.5%	-0.8%	-2.8%	-6.1%
Total	11041	11188	11263	11279	11250	11187	11101
% change	0.0%	1.3%	2.0%	2.2%	1.9%	1.3%	0.5%

Figure 6- Percentage changes in total and working age population under the base scenario



By 2034 the total population is expected to be 11.1 million people, which is an increase of 0.54%. However, the group 16-64 falls to 6.894 million, a fall of 6.1% since the year 2004. These exogenously generated population changes are fed into the economic model as explained above. The table below presents changes in the main macroeconomic variables.

Table 3 - The impact of the BASE demographic projections on Greek aggregate economic indicators

	2004	2009	2014	2019	2024	2030	2034
GDP	0.00	-0.07	-0.34	-0.79	-1.37	-2.20	-2.82
Real wage	0.00	0.65	1.18	1.62	1.97	2.30	2.48
Unemployment	9.12	8.62	8.22	7.91	7.67	7.46	7.34
Employment	0.00	-0.16	-0.52	-1.04	-1.68	-2.58	-3.23
Export Price Index	0.00	0.09	0.16	0.21	0.26	0.32	0.35
CPI	0.00	0.08	0.01	-0.04	-0.07	-0.07	-0.09

As we expect from the theoretical discussion, employment falls, in this case by 3.23% in 2034, with a corresponding decline in GDP of a little less at 2.82%. Note that because of the linearisation of the change in the working age population in the

model, the results shown in Table 1 will overestimate the reduction in the initial years, where an increase in output and employment is expected, but underestimate the rate (but not the level) of decline in the latter period of the simulation where working age population is falling.

One thing to notice is that the fall in employment is lower than the fall in working age population. Working age population falls by 6.13% whilst employment only declines by 3.23%, implying an increase in the participation rate and a fall in the unemployment rate to partially offset the negative supply side impacts. This tightening of the Greek labour market is apparent as we see increases in nominal and real Greek take home wage of 2.40% and 2.48% respectively by the year 2034.⁷

GDP and employment follow a very similar pattern as shown in Figure 7. GDP falls both because of supply and demand effects. On the demand side although we have an increase in government expenditure, which is applied in some sectors, both export demand and household demand fall because of the competitiveness effect and reduced labour income respectively. Remember we have assumed a high value of Armington elasticity in order to reflect the fact that the country is largely a price-taker. Changes in real and nominal wage are shown in Figure 8. Figure 9 shows the CPI and export price indices. Although with small deviations consumer prices initially increase and then fall below equilibrium. By 2034 the price of the composite exported good that has been calculated as the weighted average of the price of exported commodities rises by 0.35%. As a consequence, the demand for exported goods falls by 2.85%. From the supply side, in each sector the capital stock will adjust to changes in output demand but more slowly than the changes in employment so that the change in GDP will slightly lag the changes in total employment. There will also be a tendency for production to become more capital intensive as the nominal wage increases, so that there is some substitution of capital for labour.

⁷ There is not full pass through of the increased wages to prices, that is prices increase by less than wages, primarily because of the presence of imports from outwith Greece as elements of the consumption basket and as intermediate inputs in production.

Figure 7- Percentage Change of GDP and Employment

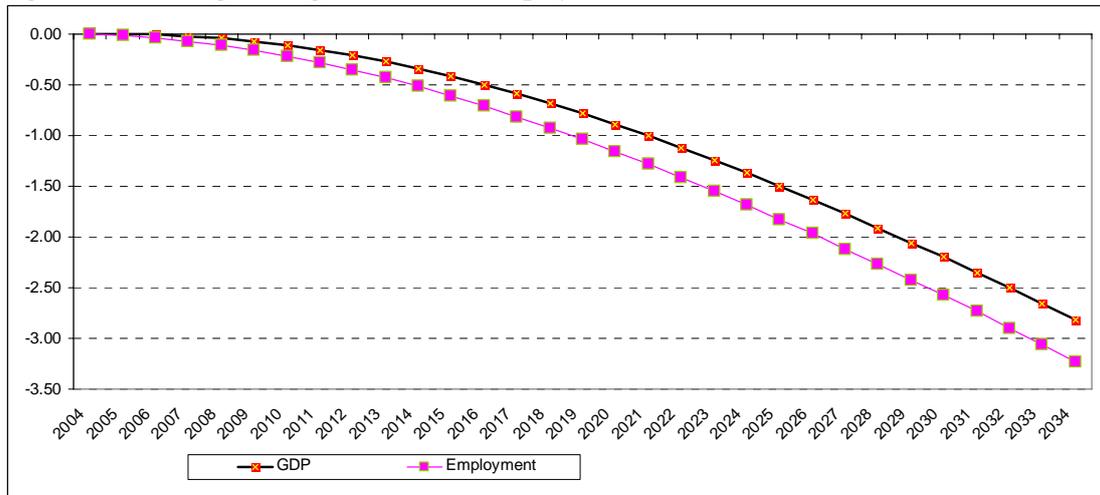


Figure 8- Percentage changes of nominal and real after tax wage

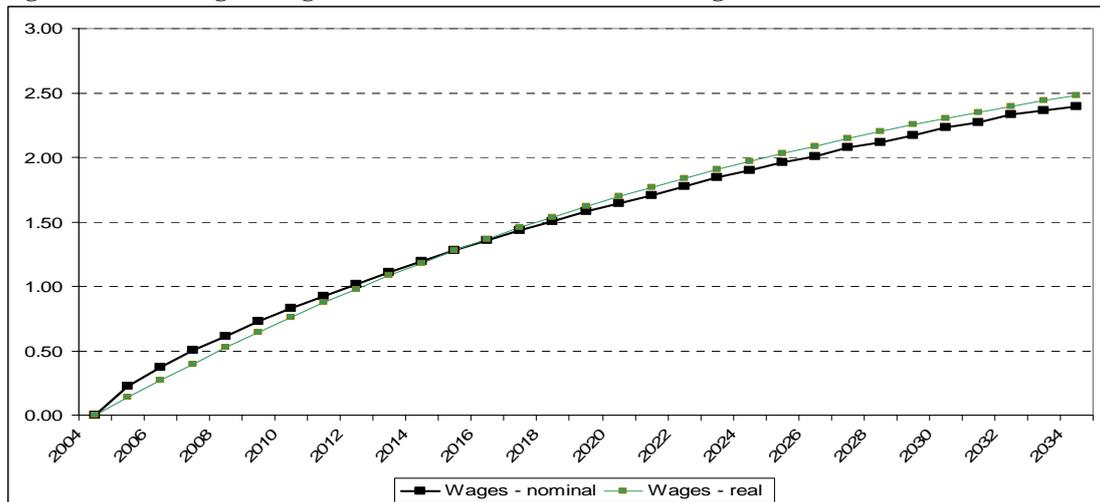
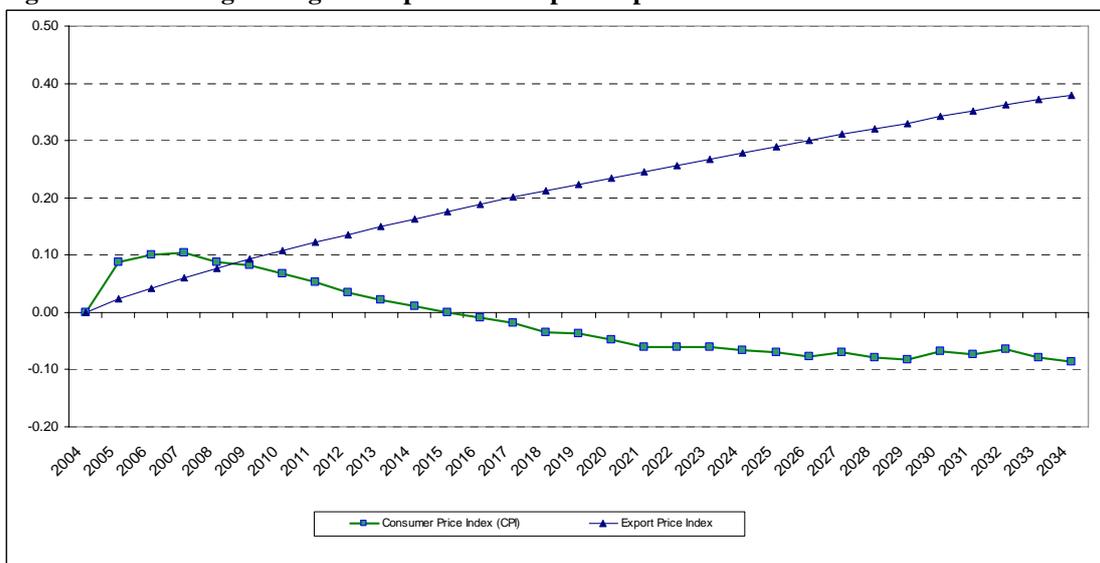


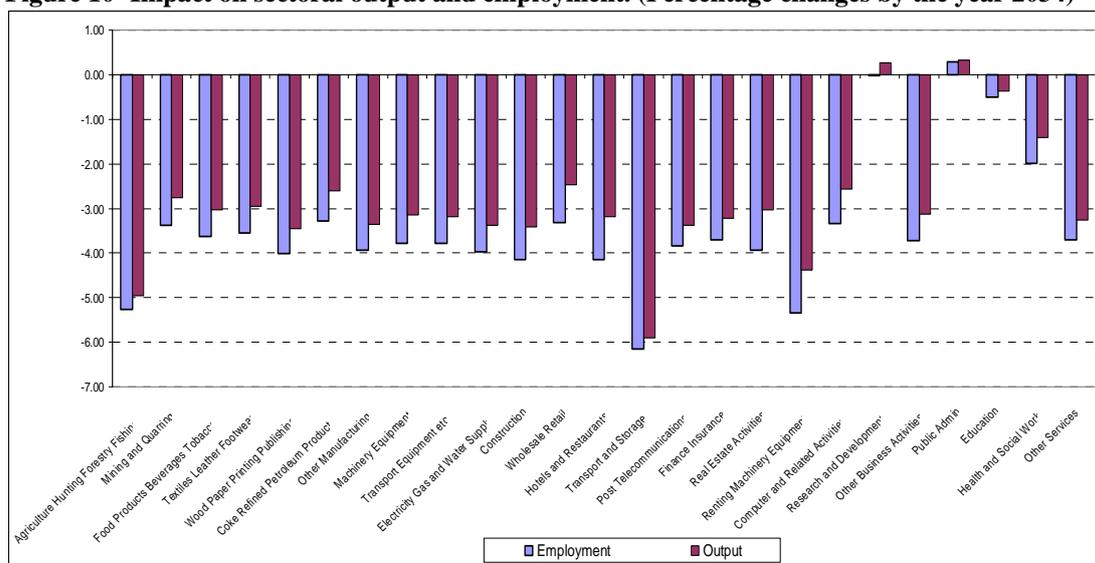
Figure 9 - Percentage changes for cpi and the exported price index



The fact that prices do not move upwards immediately is something to notice. In a simple back of the envelope model of a two good price taker economy that is presented in Appendix 3, it can be shown that in the short run prices will move up or down depending on the relative labour intensity of the exporting and non exporting sectors. It should be noted that when we run the model forward for more periods than our conventional projection horizon, and the negative supply shock becomes bigger, prices do move upwards.

Figure 10 shows the changes by 2034 in sectoral output and employment generated by the base demographic projection for Greece. At this stage we must underline once again, that we do not take into account any compositional changes in government and household demand that may spring from the changes in age composition of the population. For example, there is much discussion of the implications for health care and education provision resulting from longer life expectancy (Economic Policy Committee and European Commission, 2006). Such compositional demand changes are likely to be important but they are not the subject of this paper. These disaggregated results therefore primarily reflect more general demand-side factors. These are the extent to which the sector supplies export, investment, and household consumption or government demand.

Figure 10- Impact on sectoral output and employment. (Percentage changes by the year 2034)



By 2034 most sectors will exhibit lower employment and output than equilibrium. There is a wide variation in the impacts, ranging from an output increase of 0.3% for Public Administration output to a decrease of 5.9% for Transport and

Storage. In general those sectors selling most of their output to government demand (Public Administration, Health and Social Work, Education) are affected least because government expenditure, which in these simulations is linked to total population, remains relatively constant over the whole simulation period, and it actually increases by only 0.54% by 2034. In addition these sectors are sheltered in the sense that they are not subject to international competition.

The extent of the negative effect upon other sectors (which are mostly hit much harder) is determined by two factors. First, labour intensive sectors are worst affected because of the increased cost of labour. Second, the sectors that are more exposed to international trade feel the negative competitiveness effect more strongly. For example, sectors such as Agriculture Hunting Forestry Fishing clearly suffer these negative competitiveness effects. Transport and Storage is the sector that realises the biggest decrease. This is something that should be attributed to the high labour intensity and export propensity of the sector. However, Coke and refined Petroleum Products is labour exporting a lot (31% of output) but is not so labour intensive (7% of input goes to wages) and thus the negative impact appears to be only -2.6%. As argued above, in all sectors employment falls by more than output because as the price of labour rises firms substitute labour for capital and capital stock takes time to adjust to optimal levels.

6. Sensitivity analysis

It is well known in CGE literature that closures of markets are particularly important in the generation of results. The AMOS modelling framework allows a range of labour market closures. In order to test how sensitive the results are to variation in labour market assumptions, we conducted simulations using the base population projection under two limiting cases. The first imposes within-period fixed labour supply. This means that labour supply is a given proportion of the labour force, where the labour force is adjusted period-by-period through demographic changes. There is therefore assumed to be no unemployment or participation rate adjustment as the labour market tightens. In any individual time period, this situation is represented by a vertical labour supply curve. The second, alternative, assumption is that there is so much slack in the local labour market that an increase in labour demand is met by a change in employment but no change in the real wage. In this case, in each time period the labour supply curve would be horizontal and employment adjusts to

changes in labour demand through changes in the unemployment and participation rates. In the fixed labour supply case we expect the employment reduction and the wage increase to be larger than under our base case. In the fixed real wage scenario, the opposite results hold: we expect the employment to fall and the wage increase to be less than in the conventional wage curve configuration.

When we attempt to run the model with a fixed real wage, we get a relatively stable GDP trend while under the fixed labour supply closure GDP follows the linear reduction that we exogenously impose to the labour force. In a similar simulation by Lisenkova et. al (2008) the model fails to solve. The problem is that in this case the model produces a negative unemployment rate. The key practical point is simply this: the population constraints implied by demographic projections, combined with fixed government expenditure per head, put upward pressure on wages and continuously reduce unemployment.

On the other hand, where labour supply is completely inelastic, the whole adjustment to the labour force contraction must come through higher wages. As we have argued already, we expect employment and output to fall by more under this labour market closure, as participation and unemployment rates are not allowed to adjust. This will be combined with an even greater pressure upon wages. The figure below shows GDP trends for the different labour market closures. In Table 3 I present the percentage changes in the main aggregate indicators for 2034 under the fixed labour supply, fixed real wage and wage curve labour market closures.

Figure 11- GDP trends under different labour market closures

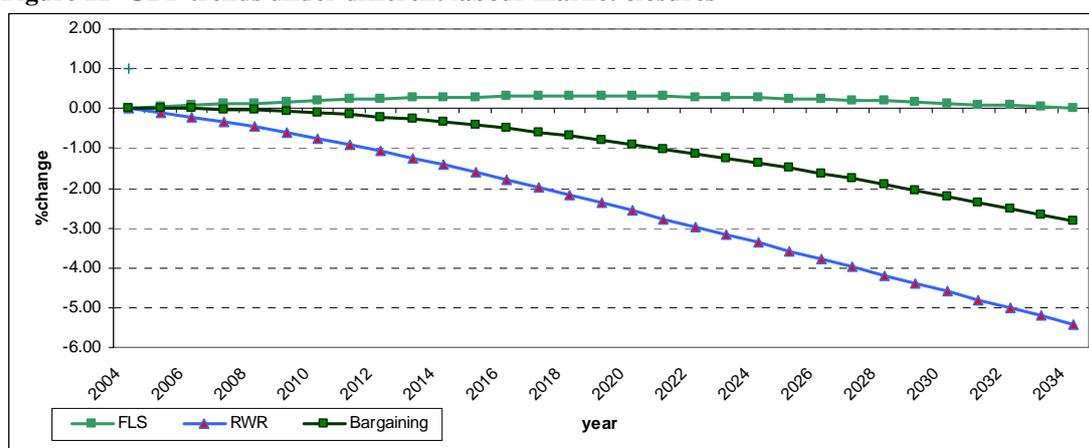


Table 4- Main aggregate indicators in 2034 under different labour market closures

	Bargaining (LNJ estimates)	Fixed Real Wage	Fixed Labour Supply
Gross Domestic Product	-2.82	-0.01	-5.41
Employment	-3.23	0.14	-6.13
Unemployment	7.34	5.30	9.12
Consumer Price Index	-0.09	-1.08	1.00
Real after tax consumption wage	2.48	0.00	4.11

Indeed our results are consistent with what economic theory suggests. Employment with fixed labour supply is expected to fall by 6.13%, exactly in line with the reduction in the labour force (and working age population), whilst under the bargaining closure the employment reduction is only -3.23%. In the case of fixed real wage we have a minor change of 0.14%, which is generated, by the demand changes that I have exogenously imposed. The corresponding figures for GDP are -5.41%, -2.82% and -0.01% respectively. The wage rate shows a reduction of an increase of 2.48% under the Bargaining closure and a 4.11% increase under fixed labour supply. By definition there is no change under the fixed real wage closure. Unemployment falls to 7.34% (from 9.12%) under Bargaining while under fixed real wage it lands at 5.30% as the labour market is purely quantity clearing. By definition, under fixed labour supply, the unemployment rate remains unaltered.

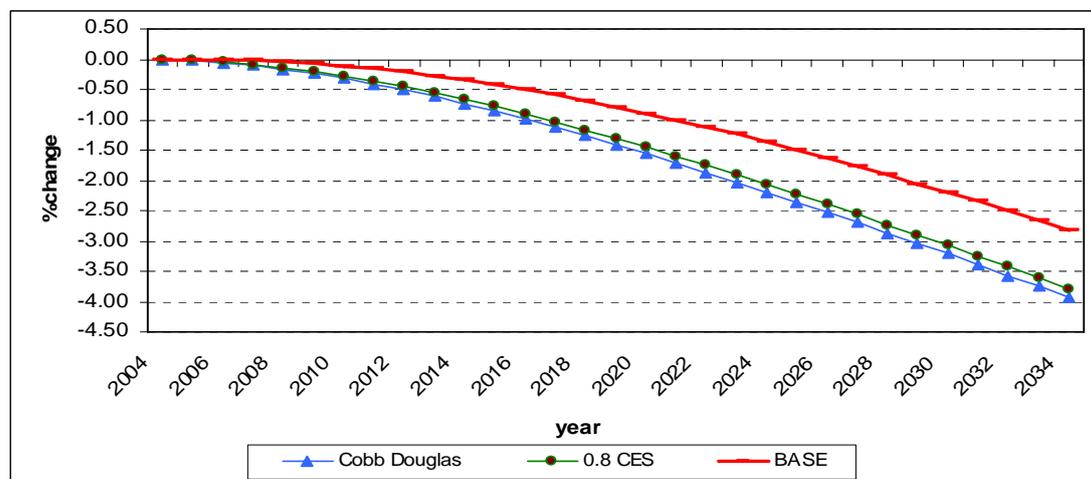
We have also tested how sensitive these economic results are to changes in the value of capital –labour elasticity as well as to changes in the value of Armington elasticity. The values we have imposed for the elasticity of substitution between capital and labour are 0.8 and 1, which corresponds to the Cobb-Douglas case. For the Armington elasticity we have imposed 2.0, which is the value that is most commonly used, and 5.0. Recall that the value imposed in the analysis of the impacts of the base scenario is 8.0. The table below presents the changes in the main aggregate indicators under each scenario.

Table 5- Main Aggregate Indicators by 2034 under different values of elasticity of substitution between local and imported goods and elasticity of labour and capital substitution

	(BASE) 0.3 CES Arm.=8	(CD) 1 CES Arm.=8	0.8 CES Arm.=8	0.3 CES Arm.=5	0.3 CES Arm.=2
GDP	-2.82	-3.93	-3.79	-2.69	-2.30
Real wage	2.48	1.26	1.43	2.60	2.94
Unemployment	7.34	8.17	8.05	7.27	7.06
Employment	-3.23	-4.58	-4.38	-3.11	-2.77
Export Price Index	0.35	0.53	0.51	0.53	1.12
CPI	-0.09	0.56	0.49	0.17	1.03

The expectation is, that with the imposition of a negative labour supply shock employment will fall more rapidly in the cases where substitutability between capital and labour is easier as firms lay off workers in the light of increased wages. Indeed employment falls by more under the Cobb-Douglas assumption by less when we have elasticity of substitution equalling 0.8 and even by less with the value of 0.3. Reversely, the wage rate increases by more the lower the substitutability between capital and labour. When laying off workers is not easy more pressure falls upon wages. In particular by the year 2034 employment falls by 4.58%, 4.38% and 3.23% for the values of unity (Cobb-Douglas), 0.8 and 0.3 respectively. The corresponding figures for GDP are 3.93%, 3.79%, 2.82% respectively. When looking at real wage, not surprisingly the ranking is reversed and the corresponding impacts are 1.26%, 1.43%, 2.48%. The figure below, shows GDP trajectories for different degrees of substitutability.

Figure 12 - GDP trends under different values of elasticity of substitution between capital and labour

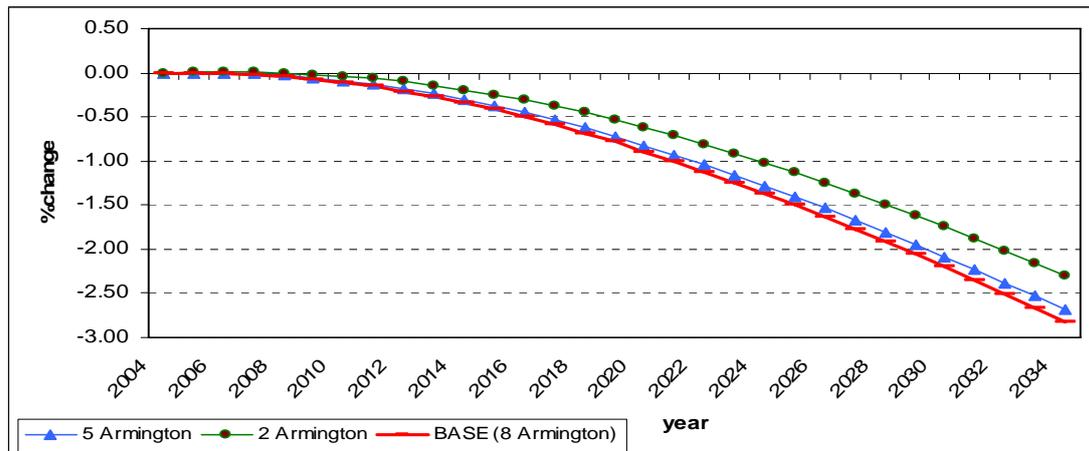


It should be noted that although the results are obviously depending on the values of elasticities, the magnitude of the variation can be considered of minor importance in relation to the variation that is caused by varying demographic parameters as we shall see in the next section.

We have also imposed different values of elasticities for the substitution of local and foreign produced goods. In this case, the higher the value of elasticity, the easier it is for firms to substitute local with foreign inputs. Thus, the more dampening the impact will be for the local economy. As the labour market tightens, producing goods locally becomes more expensive and local goods are substituted by imports. The easier this “switch” the worse the news for the local economy’s employment and GDP. Alternatively, the wage rate increases by more the lower the substitutability between local and imported goods. When import substitution is difficult firms insist on producing locally. Finally, we expect that the higher the value of elasticity the less the price of exported goods (inverse of competitiveness) will deviate from zero. In fact imposing elasticity to equal infinity would be equivalent to assuming that the law of one price holds. The values of elasticities we have imposed are 2, 5 and 8.

Indeed, employment falls by 2.77% when the value of elasticity is 2, by 3.11% when elasticity is 5 and 3.23% when it is 8. For GDP the corresponding figures are 2.30%, 2.69% and 2.82% respectively. As expected the ranking of impacts is again reversed when one looks at the behaviour of real wages and price of exported goods. Hence the corresponding increases of real wage are 2.94%, 2.60% and 2.48% respectively. The figures for competitiveness are 1.12%, 0.53% and 0.35% with the impact of elasticity being even clearer. Again, as before the magnitude of deviations in the results is small and one can claim that the values of these elasticities are not the factors that generate the results. The figure below shows the trajectories of GDP under different assumptions.

Figure 13 - GDP trends under different values of elasticity of substitution between local and imported goods



7. Variant demographic scenarios

Having checked the sensitivity of results to different labour market closures and key elasticity values we turn to assessing the sensitivity of results to varying demographic parameters. These parameters are the fertility rate, the male and female life expectancy, and the net-migration rate. The tables below summarise the combination of assumptions that is used in each scenario and give the projected changes of total and working age population by 2034 in each case.

Table 6 – Combination of assumptions for each demographic scenario

Scenario	Fertility	Life expectancy	Migration
1 Base projection	Base	Base	Base
2 High fertility	High	Base	Base
3 Low fertility	Low	Base	Base
4 High life expectancy	Base	High	Base
5 Low life expectancy	Base	Low	Base
6 High migration	Base	Base	High
7 Low migration	Base	Base	Low

Table 7 – Percentage changes of total and working age population by 2034 under different scenarios

	BASE Scenario	Low Expectancy	High Expectancy	Low fertility	High Fertility	Low Migration	High Migration
16-64	6894	6876	6917	6813	6989	6394	7394
%	-6.13%	-6.37%	-5.81%	-7.2%	-4.8%	-12.94%	0.68%
Total	11101	10993	11234	10849	11436	10435	11768
%	0.5%	-0.4%	1.8%	-1.7%	3.6%	-5.5%	6.6%

Comparing the columns of table 6 we see that the variation in net-migration is the demographic variable that causes the greater variation in the size of total and working age population and thus in size of shocks introduced into the economic model. The amount of net migration assumed in the “low migration” scenario is 20,000 per year while in the “high migration” scenario it is 60,000 per year. The values of fertility and life expectancy are the ones that are assumed by ESYE in the corresponding scenarios and presented in section 5. In particular, assuming high and low life expectancy results in GDP being 2.57% and 2.99% below base respectively by 2034. The corresponding figures when we vary fertility are -1.94% and 3.55%. However, the discrepancy of results between the scenarios of “high” and “low” net-migration is much bigger. Assuming “low” (20,000) net-migration results in GDP being 6.86% below base, but assuming “high” migration results in GDP being *above base by 0.88%*. The paramount importance of net-migration has led us to generate further scenarios by varying even more the amount of net-migration while holding the other two variables constant. We generate a scenario with zero net migration as well as scenarios that involve 80,000 and 100,000 annual net-migration. The results summarised in the table and the figures below are illuminating and the positive impact of in-migration is very apparent. Note that in the scenario where we assume zero net migration (natural change only) GDP falls by 11.33% in 2034.

Table 8 – Main aggregate economic indicators for different net-migration scenarios

	2004	2009	2014	2019	2024	2029	2034
<i>BASE</i>							
GDP	0.00	-0.07	-0.34	-0.79	-1.37	-2.06	-2.82
Real wage	0.00	0.65	1.18	1.62	1.97	2.25	2.48
Unemployment	9.12	8.62	8.22	7.91	7.67	7.49	7.34
Employment	0.00	-0.16	-0.52	-1.04	-1.68	-2.42	-3.23
Export Price Index	0.00	0.09	0.16	0.21	0.26	0.31	0.35
CPI	0.00	0.08	0.01	-0.04	-0.07	-0.08	-0.09
<i>Zero</i>							
Gross Domestic Product	0.00	-0.61	-1.9	-3.74	-5.98	-8.54	-11.33
Real wage	0.00	1.88	3.57	5.1	6.47	7.7	8.83
Unemployment	9.12	7.74	6.68	5.87	5.24	4.73	4.31
Employment	0.00	-0.98	-2.63	-4.77	-7.27	-10.04	-12.99
Export Price Index	0.00	-0.09	-0.11	-0.07	0.02	0.15	0.32
CPI	0.00	-1.01	-1.38	-1.56	-1.6	-1.52	-1.37
<i>Low Migration (20,000)</i>							
GDP	0.00	-0.34	-1.1	-2.2	-3.58	-5.15	-6.86
Real wage	0.00	1.24	2.31	3.23	4.02	4.69	5.28
Unemployment	9.12	8.18	7.45	6.88	6.44	6.08	5.79
Employment	0.00	-0.56	-1.54	-2.83	-4.35	-6.05	-7.87
Export Price Index	0.00	0	0.02	0.06	0.12	0.2	0.29
CPI	0.00	-0.47	-0.7	-0.83	-0.88	-0.87	-0.82
<i>High Migration (60,000)</i>							
GDP	0.00	0.18	0.37	0.53	0.67	0.78	0.88
Real wage	0.00	0.08	0.16	0.21	0.23	0.24	0.24
Unemployment	9.12	9.06	9.00	8.96	8.94	8.93	8.93
Employment	0.00	0.22	0.44	0.62	0.77	0.9	1.01
Export Price Index	0.00	0.18	0.31	0.39	0.44	0.47	0.48
CPI	0.00	0.64	0.75	0.8	0.81	0.81	0.78
<i>Eighty</i>							
GDP	0.00	0.43	1.05	1.77	2.56	3.41	4.31
Real wage	0.00	-0.46	-0.78	-1.04	-1.26	-1.45	-1.61
Unemployment	9.12	9.5	9.77	10	10.2	10.37	10.52
Employment	0.00	0.59	1.35	2.18	3.05	3.97	4.92
Export Price Index	0.00	0.28	0.47	0.6	0.66	0.69	0.68
CPI	0.00	1.23	1.51	1.67	1.76	1.78	1.76
<i>Hundred</i>							
GDP	0.00	0.66	1.69	2.93	4.33	5.86	7.49
Real wage	0.00	-0.97	-1.63	-2.14	-2.54	-2.86	-3.13
Unemployment	9.12	9.94	10.55	11.05	11.45	11.79	12.08
Employment	0.00	0.94	2.2	3.63	5.17	6.82	8.54
Export Price Index	0.00	0.38	0.65	0.82	0.92	0.95	0.95
CPI	0.00	1.81	2.29	2.6	2.76	2.83	2.82

Figure 14 - GDP % change under different scenarios for the amount of annual migration

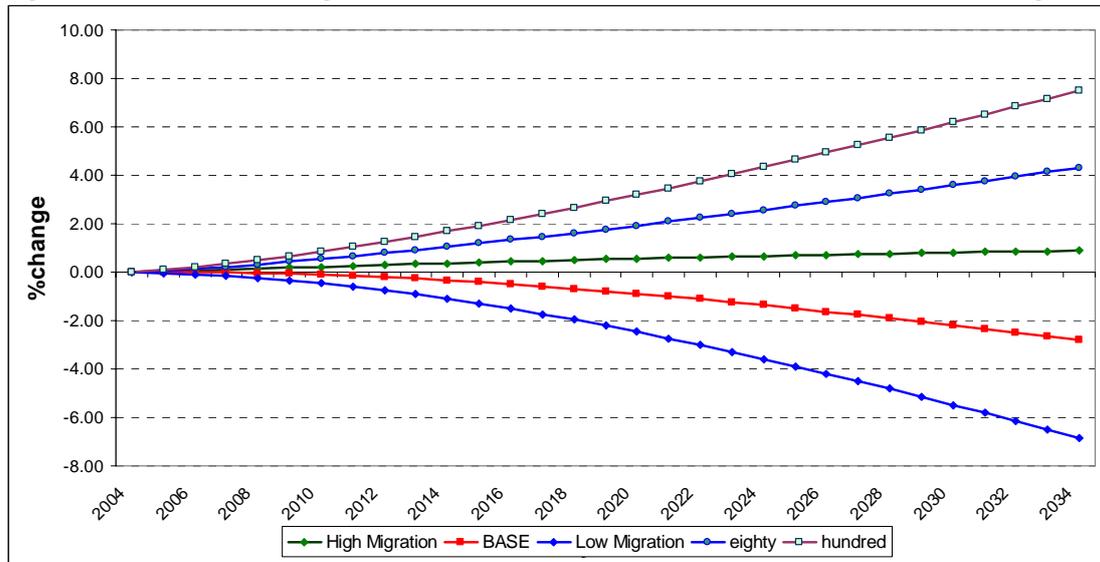
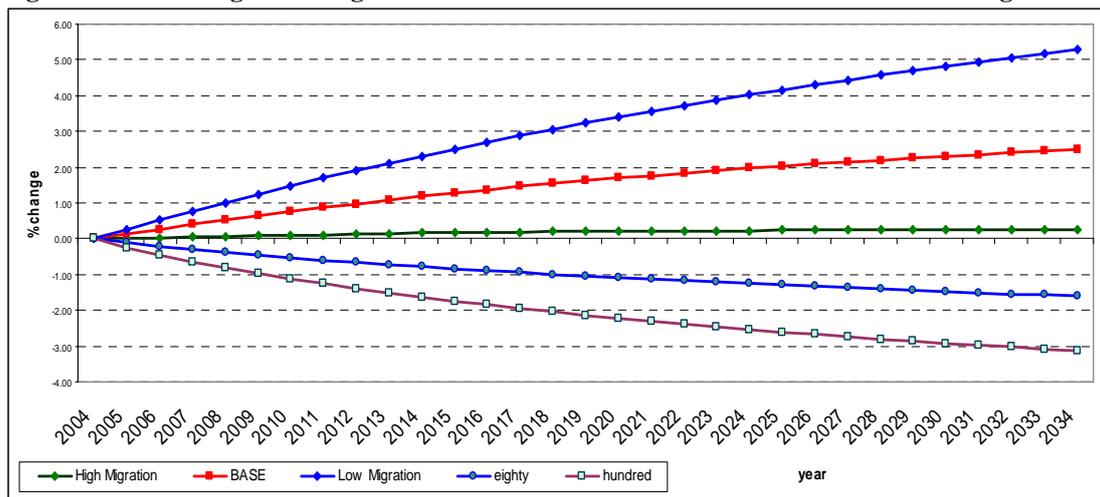


Figure 15 – Real wage % change under different scenarios for the amount of annual migration



The importance of net-migration stems from the fact that migrants due to their age structure have an immediate impact upon the size of the working age population as a great deal of them is within the 16-64 age-group. The thing to notice is that the higher the level of migration the higher the GDP and lower the real wage. The result to highlight is that only with scenarios that involve 60,000 positive net-migration per annum or more will the GDP remain above equilibrium levels. In this case the real wage remains practically stable as the level of net-migration is completely off setting the tightening of the labour market.

The results of the economic model are consistent with the theoretical framework, which we put in place in Section 2 and they are mainly driven by the labour market and the subsequent impact on competitiveness. Figure 15 shows the real wage change

associated with the different migration scenarios. In this case, the 60,000 in-migration case produces a negligible change in the real wage. We know that in this simulation, the working age population is rising, but more slowly than the total population. In terms of the analysis in Section 2, the outward shift of the labour demand curve, as government expenditure rises in line with total population, is greater than the outward shift in the labour supply curve as the working age population increases. There is therefore an increase in employment but also a small increase in the wage. However, for the examples where in-migration is less than 60,000, the wage increase is much greater and is not accompanied by any expansion in government expenditure.

8. Conclusions

This paper attempts to explore the consequences of projected demographic developments in Greece. We have interacted a demographic model (FIV-FIV) and an economic model G-AMOS which is an economic model for the Greek economy. We conclude that if current demographic trends continue, that is under what we call the “base scenario” there will be a negative impact upon economic activity, which will stem from a tightening of the labour market. GDP falls by 2.82% while the real wage increases by 2.48% in the next three decades. Employment also falls by 3.23%.

The demographic variable with the greatest importance appears to be the level of annual net-migration. Migrants are mainly of younger ages and are often accompanied with children, and thus their presence tends to mitigate the tendency for population ageing. The importance of net-migration is apparent in the results of the “zero net-migration” scenario. This scenario answers the following question. What would happen if Greek borders close completely and the Greek population changes depend only on the underlying birth rate and life expectancy? This is the “worst” scenario of all the ones we have examined and it reveals to a great extent the positive impact that migrants have had and will continue to have in the years to come. According to this scenario, GDP and employment fall by 11.33% and 12.99% respectively while the real wage increases by 8.33%.

Remember that in the course of this paper we have not examined, the impacts of a change in the composition of public and private demand that may occur due to a change in the demographics, neither we have explored the impact of the likely

changes of the overall productivity that is associated with the presence of younger and older workers in the labour market.

In any case, the apparent policy implication is that the level of annual net-migration is of paramount importance for the demographic future of Greece and that the current levels of migration are not sufficient to completely offset the negative impacts of ageing. In our simulation exercise there is no distinction between legal and illegal migrants. Full legalisation of existing foreign population could be the first step towards a consistent and viable migration policy. Legalisation would increase the proceeds of national insurance and income tax while it could also reduce remittances and the leakage of funds outside the country (Kontis et. al., 2006). An active migration policy thereafter is necessary in order to ensure sufficient flows of migration. However, as migrants do grow old themselves they cannot be relied upon forever. Family friendly policies that could enhance fertility rates as well as non-discrimination against older workers in the labour market could ease the problems generated by the existing demographic trends.

There are of course a number of issues that have not been explored in this paper. First of all there are demand effects of demographic change. In addition, if we believe that younger and older workers have different productivities there will be an impact due to changes in the overall productivity of the labour force. Also, if labour force participation is age dependant the size of the labour force will be affected in a different way than the one assumed in this paper. Finally, if ageing is defined in a different way as Shoven (2007) the whole discussion will have new fundamentals. The current configuration of G-AMOS allows us to explore the questions mentioned above.

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Appendix 1: The Sectoral Disaggregation in AMOS

Table A1.1 Sectors (Activities/Commodities) Identified in AMOS

	Sector Name	ISIC
1	<i>AGRICULTURE, HUNTING, FORESTRY AND FISHING</i>	01-05
2	<i>MINING AND QUARRYING</i>	10-14
3	<i>FOOD PRODUCTS, BEVERAGES AND TOBACCO</i>	15-16
4	<i>TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR</i>	17-19
5	<i>WOOD, PAPER, PRINTING, PUBLISHING</i>	20-22
6	<i>COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL</i>	23
7	<i>OTHER MANUFACTURING</i>	24-28
8	<i>MACHINERY, EQUIPMENT</i>	29-33
9	<i>TRANSPORT EQUIPMENT ETC.</i>	34-37
10	<i>ELECTRICITY, GAS AND WATER SUPPLY</i>	40-41
11	<i>CONSTRUCTION</i>	45
12	<i>WHOLESALE AND RETAIL TRADE; REPAIRS</i>	50-52
13	<i>HOTELS AND RESTAURANTS</i>	55
14	<i>TRANSPORT AND STORAGE</i>	60-63
15	<i>POST AND TELECOMMUNICATIONS</i>	64
16	<i>FINANCE, INSURANCE</i>	65-67
17	<i>REAL ESTATE ACTIVITIES</i>	70
18	<i>RENTING OF MACHINERY AND EQUIPMENT</i>	71
19	<i>COMPUTER AND RELATED ACTIVITIES</i>	72
20	<i>RESEARCH AND DEVELOPMENT</i>	73
21	<i>OTHER BUSINESS ACTIVITIES</i>	74
22	<i>PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY</i>	75
23	<i>EDUCATION</i>	80
24	<i>HEALTH AND SOCIAL WORK</i>	85
25	<i>OTHER SERVICES</i>	90-99

Appendix 2: The AMOS Model

Table A2.1: A Condensed Version of the AMOS CGE Model

1. Commodity Price	$p_i = p_i(w_n, w_{k,i})$
2. Consumer Price Index	$cpi = \sum_i \theta_i p_i + \sum_i \theta_i^{REU} \bar{p}_i^{REU} + \sum_i \theta_i^{ROW} \bar{p}_i^{ROW}$
3. Capital Price Index	$kpi = \sum_i \gamma_i p_i + \sum_i \gamma_i^{REU} \bar{p}_i^{REU} + \sum_i \gamma_i^{ROW} \bar{p}_i^{ROW}$
4. User Cost of Capital	$uck = uck(kpi)$
5. Wage Equation	$w_n = w_n(N, \bar{L}, cpi)$
6. Capital Stock	$K_{i,t}^S = (1 - d_i)K_{i,t-1} + \Delta K_{i,t-1}$
7. Labour Demand	$N_i^D = N_i^D(Q_i, w_n, w_{k,i})$
8. Capital Demand	$K_i^D = K_i^D(Q_i, w_n, w_{k,i})$
9. Labour Market Clearing	$\sum_i N_i^D = N$
10. Capital Market Clearing	$K_i^S = K_i^D$
11. Household Income	$Y = \Psi_n N w_n + \Psi_k \sum_i K_i w_{k,i}$
12. Commodity Demand	$Q_i = C_i + I_i + G_i + X_i$
13. Consumption Demand	$C_i = C_i(p_i, \bar{p}_i^{REU}, \bar{p}_i^{ROW}, Y, cpi)$
14. Desired Capital Stock	$K_i^* = K_i^D(Q_i, w_n, uck)$
15. Capital Stock Adjustment	$\Delta K_i = \lambda_i (K_i^* - K_i)$
16. Investment Demand	$I_i = I_i\left(p_i, \bar{p}_i^{REU}, \bar{p}_i^{ROW}, \sum_j b_{i,j} \Delta K_j\right)$
17. Government Demand	$G_i = \varphi_i p_i \bar{P}$
18. Export Demand	$X_i = X_i\left(p_i, \bar{p}_i^{REU}, \bar{p}_i^{ROW}, \bar{D}^{REU}, \bar{D}^{ROW}\right)$

NOTATION

Activity-Commodities

i, j are activity/commodity subscripts.

Transactors

EU = European Union, ROW = Rest of World

Functions

$p(\cdot)$	cost function
$w_n(\cdot)$	wage equation
$uck(\cdot)$	user cost of capital formulation
$K^D(\cdot), N^D(\cdot)$	factor demand functions
$C(\cdot), I(\cdot), X(\cdot)$	Armington consumption, investment and export demand functions, homogenous of degree zero in prices and one in quantities

Variables

C	consumption
D	exogenous export demand
G	government demand for local goods
I	investment demand for local goods
ΔK	investment demand by activity
K^D, K^S, K^*, K	capital demand, capital supply, desired and actual capital stock
L	labour force
N^D, N	labour demand and total employment
P	population
Q	commodity/activity output
X	exports
Y	household nominal income
b	elements of capital matrix
cpi, kpi	consumer and capital price indices
d	physical depreciation
p	price of commodity/activity output
t	time subscript
uck	user cost of capital
w_n, w_k	wage, capital rental
Ψ	share of factor income retained in region

θ	cpi weights
γ	kpi weights
φ	government expenditure coefficient
λ	capital stock adjustment parameter

Notes:

Variables with a bar are exogenous.

A number of simplifications are made in this condensed presentation of AMOS

1. Intermediate demand is suppressed throughout e.g. only primary factor demands are noted in price determination in equation (1) and final demands in the determination of commodity demand in equation (12).
2. Income transfers are generally suppressed.
3. Taxes are ignored.
4. There are implicit time subscripts on all variables. These are only stated explicitly in the capital updating equation (6).

Appendix 3: A “back of the envelope model” - Prices in a Cobb-Douglas World with two goods

E= export sector, D= domestic sector

α = labour intensity in export sector

β = labour intensity in domestic sector

Fixed non labour inputs in both export and domestic sector

All non labour income goes outwith the region

Export sector faces parametric world price = numeraire

Constant share of wage income is spent on domestic output

Production

$$(1) Q_E = AL_E^\alpha$$

$$(2) Q_D = BL_D^\beta$$

Distribution

$$(3) \frac{wL_E}{1 \times Q_E} = \alpha$$

$$(4) \frac{wL_D}{p_D Q_D} = \beta$$

Demand

$$(5) p_D Q_D = \gamma Lw$$

STEP 1: Employment shares

Substitute (5) in (4) and rearrange

$$(6) L_D = \gamma\beta L$$

Total labour force

$$(7) L = L_E + L_D$$

Substituting (7) in (6)

$$(8) L_E = (1 - \gamma\beta)L$$

$$(9) \dot{L}_E = \dot{L}_D = \dot{L}$$

The dot notation implies proportional changes

STEP 2: Wage determination

Substitute equation (1) into (3) and rearrange.

$$(10) \quad w = \frac{\alpha A}{L_E^{1-\alpha}}$$

Which gives (using equation 9)

$$(11) \quad \dot{w} = -(1-\alpha)\dot{L}_E = -(1-\alpha)\dot{L}$$

A fall in the labour supply increases the wage in the export goods sector (relative to fixed export prices)

STEP 3: Determination of price of domestic good

Substitute (2) into (4) and rearrange

$$(12) \quad \frac{w}{p_D} \times \frac{L_D}{BL_D^\beta} = \beta \quad \Rightarrow \quad w = \frac{p_D \beta B}{L_D^{1-\beta}}$$

$$(13) \quad \dot{w} = \dot{p}_D - (1-\beta)\dot{L}_D = -(1-\beta)\dot{L}$$

Substitute equation (11) into (13)

$$(14) \quad \dot{p}_D - (1-\beta)\dot{L} = -(1-\alpha)\dot{L}$$

$$(15) \quad \dot{p}_D = (a-\beta)\dot{L}$$

Therefore the price of the domestic good will rise (relative to the export good) when labour supply is reduced if $\beta > \alpha$. (i.e. the labour intensity of the domestic sector is greater than the labour intensity of the traded sector.)

STEP 4: Change in CPI

If all income is consumed, γ is spent on domestic goods whose price is parametric

$$cpi = \gamma \dot{p}$$

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