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'ECONOMIC GEOMETRY': MARSHALL'S AND OTHER EARLY REPRESENTATIONS OF DEMAND AND SUPPLY.

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‘ECONOMIC GEOMETRY’: MARSHALL’S AND OTHER EARLY REPRESENTATIONS OF DEMAND AND SUPPLY

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ABSTRACT

Does an apparent (minor) anomaly, said to occur not infrequently in elementary expositions of supply and demand theory, really imply – as seems to be suggested – that there is something a bit odd about Marshall’s diagrammatic handling of demand and supply? On investigation, we find some interesting differences of focus and exposition amongst the theorists who first developed the ‘geometric’ treatment of demand and supply, but find no reason, despite his differences from other marginalist pioneers such as Cournot, Dupuit and Walras, to consider Marshall’s treatment either as unconventional or forced, or as to regard him as the ‘odd man out’.

Introduction

In the standard textbooks, introductory discussions of demand and supply normally treat quantities demanded and supplied as functions of price (rather than *vice versa*), and complement that discussion with diagrams in the standard format, showing price on the vertical axis and quantities demanded and supplied on the horizontal axis. No references need be cited. Usually this presentation is accepted without comment, but it can happen that a more numerate student observes that something of an anomaly appears to exist – in that the diagrams show price, which,

[1] Roy’s thanks go to Darryl Holden who raised the question about Marshall's diagrams, and for his subsequent advice, and to Eric Rahim, as always, for valuable comment.

in the text, has been presented as the *independent* variable, on the *ordinate* (vertical axis) and the dependent variable (quantity demanded or supplied) on the abscissa (horizontal axis), rather than, as would accord with mathematical convention, with the axes transposed (measuring the independent variable along the x-axis).

If raised, the point is likely to be dismissed with an airy reassurance that this is just how economists do these things, or perhaps with a reference to the fact that this was the diagrammatic convention adopted by Alfred Marshall, the great English pioneer of supply and demand analysis, and that the profession has subsequently, and unquestioningly, followed his lead.

Although there does seem to be something a little peculiar here, this oddity or anomaly has not attracted much attention. However, A. K. Whitaker, an authority on Marshall, has commented on the matter. In suggesting an explanation, he focuses on the Marshallian inheritance (Whitaker, 1987):

> It is probably due to Marshall's influence that English-speaking economists still graph demand and supply curves with quantity on the horizontal axis even though adopting a more Walrasian perspective which treats quantities demanded and supplied as functions of market price.

And he remarks further:

> Marshall recognized (1920, p.457n) that it would be more natural when dealing with market demand to view quantity as a function of price, as Cournot (1838, pp.44-55) had done, but he chose the converse approach to maintain symmetry with his treatment of supply. Believing in the importance of scale economies in production, he deemed it generally impossible to treat quantity supplied per unit of time as a single-valued function of market price. Instead, adopting what he took to be the businessman's perspective, he introduced the concept of 'supply price', the minimum uniform price at which any given quantity will be supplied in the market.

Taking Whitaker's two comments together, it appears that, firstly, he attributes the anomaly in question to the general adoption, by 'English-speaking economists' of Marshall's diagrammatic treatment, whereby quantity is measured along the horizontal axis, despite that depiction being

inconsistent with the usage common in discussion (which is to treat price as the independent variable) and, secondly, he is hinting that Marshall himself was not altogether happy with his own diagrammatic representation, and might even have preferred the 'Continental' alternative, *a la* Cournot.

Why then, we may ask, did Marshall choose a different mode of diagrammatic presentation from that adopted by other pioneers of marginalist theory, as, for instance, Cournot, Dupuit and Walras? Whitaker seems to imply that Marshall was out on a limb, and not really because he wanted to be there. Is there then something odd about the manner in which the Marshallian diagram, to which we have become so accustomed, is drawn? Could it have been because Marshall flouted mathematical convention that this anomaly arises?

While we readily admit these are hardly issues of life and death, there does seem to be some fogginess around them: if we are to clarify the picture we need to know just how Marshall and other early exponents of 'economic geometry' handled diagrams of demand and supply, and we need to work out, if we can, why they chose to proceed in the ways that they did. Let us make a (brief) tour of the territory, recognising, however, that to understand the contexts out of which the various demand and supply curves emerged, some detail may at times be necessary. In order to set Marshall in context, we begin by reviewing the treatment of demand and supply favoured by marginalist pioneers other than Marshall. We then examine Marshall's practice.

Demand and supply with price on the horizontal axis

Let us consider first those early 'marginalist' authors who employed a graphical exposition, but who, unlike Marshall, drew their diagrams with price on the horizontal axis and quantity on the vertical.

It is appropriate that we start with A A Cournot (1801-1877), a highly original thinker, mathematician and philosopher, and the first economist actually to draw demand and supply curves. With respect to demand, Cournot writes $D = f(p)$, and (assuming the function to be continuous), 'takes it as an empirical proposition that it is downward-sloping (the *loi de débit*,

'law of demand') and proceeds to draw it in price-quantity space'[2]. Consistent with his treatment of price as the independent variable, he places price on the horizontal axis and quantity on the vertical. From our perspective it is relevant to note that Cournot, in sharp contrast to authors such as Gossen, Jevons and Marshall himself, categorically excluded from his analysis of demand, and construction of the demand curve, all subjective factors to which reference might have been made. His 'demand function' therefore, rather than showing a relationship between price and the quantity agents *sought to buy*, represented simply an empirical relationship between price and quantity *sold.*[3] We take it that, as a mathematician, Cournot recognised the existence of a functional relationship between price and quantity demanded, but held, again as a mathematician, that it was not his role to become involved with subjective concepts and imprecise quantities. In his own words, (1838, p.47) he wanted nothing to do with 'ideas of utility, scarcity and suitability to the needs of mankind' which he considered 'variable and by nature indeterminate, and consequently ill suited for the foundation of a scientific theory.'

Cournot's attention was initially directed to the side of demand rather than supply. He observed that as price varied, and quantity bought altered, the seller's revenue was systematically affected: in terms of his diagram, corresponding to each price on the demand curve, a price–quantity rectangle represented the revenue obtained at that selling price. This perception (together with the assumption that costs were either zero or independent of quantity produced) allowed him to identify the price which a profit-maximising monopolist would charge to maximise revenue. Cournot is famous also for having formally specified elasticity of demand as percentage change in quantity demanded over percentage change in price. A further contribution was the introduction of a supply curve depicting output as an increasing function of price, based on the assumption that marginal costs increased with output. It will be noticed that Cournot's measurement of price along the horizontal axis accords with his working out of these several 'thought experiments', in which, in each case, price is taken as the independent variable.

Jules Dupuit (1804–1866), civil engineer and economist, was not only, like Cournot, a pioneer of marginal analysis, but was also one of those who (independently) discovered the marginal utility

[2] http://cepa.newschool.edu/het/profiles/cournot.htm
[3] Ibid.

principle. He identified the concept of consumers' surplus, also the phenomenon of deadweight loss due to monopoly pricing, and investigated the implications of discriminatory pricing.

As a practical civil engineer, Dupuit's concern was – as the title of his 1844 paper has it – 'measurement of the utility of public works' – i.e. of works such as canals, railways and bridges. He appraises methods of measurement suggested by others, by for example J B Say and L Navier, and puts forward in place of their demonstrably inadequate procedures, his own proposal, which was to estimate the benefit to society by measuring the value of what, in modern terminology, we call consumers' surplus (or a change therein), as attributable to the project in question. He employed a downward-sloping demand curve, and clearly identified as the measure of consumers' surplus the area under under the demand curve and above the cost schedule. Being prepared to estimate a money value for the consumers' surplus accruing in specified circumstances, Dupuit evidently did not share Cournot's doubts about the feasibility of measuring (at least making some sort of measure of) utility.

Although Dupuit is credited with discovery of the concept of diminishing marginal utility, he did not offer any explanation of why that should be so. There is no discussion, as in Gossen or Jevons, of the 'pleasure principle', or of any sort of utility function: in his famous 1844 paper Dupuit seems simply to take it as an observed fact that willingness to pay for extra increments of a good diminishes with quantity already acquired. For example (1844/1952, p.86) he says,

> Thus, examining the facts more closely, we have come to see that the utility of everything which is consumed varies according to the person consuming it. Nor is this all: each consumer himself attaches a different utility to the same thing according to the quantity which he can consume. Thus, a purchaser who would have bought 100 bottles at10 sous might only buy 50 at 15 sous and 30 bottles at 20 sous.

Then, approaching the concept of consumers' surplus (1844/1952, p.89):

> To sum up, political economy has to take as the measure of the utility of an object the maximum sacrifice which each consumer would be willing to make to acquire the object.

By a ‘thought experiment’ he makes the point that the value of a commodity to the consumer can be measured by the whole area under the demand curve (1844/1952, p.96):

> Suppose that all these . . . commodities of which we want to discover the utilities, are all subjected to a tax which rises by small steps. Each successive increase will cause a certain quantity of our commodity to disappear from consumption. This quantity, multiplied by the rate of tax, will give its utility expressed in money. By thus letting the tax go up until there are no more consumers, and by adding together all the products of this multiplication process, we will arrive at the total utility of the goods.

He subsequently notes (1844/1952, p.97) that, if costs of production are involved, it is the area under the demand curve, net of costs, that constitutes the net (or as he calls it ‘relative’ or ‘definitive’) social value of a commodity.

Using this model, Dupuit examines the effects on social welfare of various possible events – changes in tolls or taxes, changes in costs of production; this eventually leads him back to his critique of other suggested methods of evaluation, concluding with the point that, in estimating the benefit accruing from transport improvements, such improvements should be evaluated by the extent to which they reduce overall costs of production (and not by whatever change in transport costs, as such, results).

After commenting that the variability of real-world data (‘dependent on the volatile will of human beings’), implies that ‘it is of no avail to try to determine [price-quantity relationships] exactly by experience or groping experiment’, Dupuit (1844/1952, p.103) observes however ‘that there do exist certain general laws . . . One of these laws is that consumption expands when price falls; another, that the increase in consumption due to a price fall will be greater, the lower the initial price.’

Finally, in a ‘note’ appended to his text, Dupuit explains how his demand curve is drawn (1844/1952, pp.106-107):

> If it be supposed . . . that along a [horizontal] line Op the lengths Op, Op', Op" . . . represent various prices for an article, and that the verticals pn, p'n', p"n" . . . represent the number of articles consumed corresponding to these prices, then it is possible to construct a curve Nnn'n"P which we shall call the curve of consumption. ON represents the quantity consumed when the price is zero, and OP' the price at which consumption falls to zero. . . . The utility of a natural product the acquisition of which requires no expense, is expressed by the large triangle NOP.

Given that throughout Dupuit's discussion of demand and value, every single example presented, every thought experiment conducted, takes the form of an inquiry as to the effect of price change on quantity demanded or on amount of surplus accruing to consumers, it is no cause for surprise that, in constructing his diagrams, Dupuit treats price as the independent variable, placing it on the horizontal axis. Giving priority to price in that way is consistent with his primary focus being on the impact of cost changes on social welfare; while accepting diminishing marginal utility as an empirical fact, Dupuit doesn't seem to have been interested in investigating the phenomenon more deeply, in the manner of Gossen or Jevons or of Marshall himself, or explaining to his readers its philosophical or psychological foundations.

It was H C Fleeming Jenkin[4] (1833-1885), a distinguished Edinburgh electrical engineer and inventor, who 'introduced demand and supply curves – indeed the technique of diagrammatic analysis – into the *English* [i.e. Anglo-Saxon] economic literature c1870' (Humphrey, 1992, p.14). (Marshall, as reported by Edgeworth, was much chagrined when Professor Foxwell drew his attention to Jenkin's 1870 article shortly after its publication.)

Jenkin formulated (in 1868) demand and supply functions, $D = f(A + 1/x)$ and $S = f(B + x)$, where x = price and (in 1870) represented these equations in graphical form (Jenkin, 1877/1931). Like Cournot and Dupuit he showed price on the horizontal axis and quantity on the vertical[5]. He interpreted, as Dupuit had done, the inverse functional relationship between price and demand as reflecting the fact that lower price compensated for diminishing marginal utility as quantity

[4] Jenkin's unusual first name is correctly pronounced 'Flemming'; he was named after an admiral, 'one of his father's protectors – his father was an officer - in the navy' (Source: Robert Louis Stevenson (1887/1924, p.21).

[5] While Jenkin clearly had his own good reason (given the theses he was presenting) for placing price on the vertical axis, he may by then also have had, as a model, Dupuit's diagrams; Eklund and Hébert note (1999, p.348) that Jenkin evidently came across Dupuit's work 'sometime between 1868 and 1871'.

purchased increased. He proposed both short and a long-run supply curves, each upward-sloping: in the short-run quantity supplied was said to rise with price, the price increase drawing an increasing proportion of stocks on to the market; in the long-run higher prices bid resources away from alternative uses, so permitting production to increase. Jenkin, using his demand-supply diagram, conducted other 'thought experiments', demonstrating that, as he supposed, higher wages achieved by trade unions would restrict employment, and also how the burden of a tax-caused price rise is distributed between consumers and producers.

It would seem therefore that Jenkin's diagrammatic representation of demand and supply with price on the horizontal axis was perfectly in accord with his analytical focus (in the manner of Dupuit) on the consequences of changes in price.

Leon Walras (1834-1910), with mathematical expertise attributable to his training as an engineer, was one of the great original thinkers who brought about the 'marginal revolution'. He is famous particularly for pioneering the development of neoclassical general equilibrium theory.

His attention was directed to investigating the possibility of attaining market-clearing equilibrium in a multi-market economic system; he proceeded by modelling the behaviour of optimising agents deciding quantities to buy or sell, given the prices they face in competitive markets. In getting down to serious analytical business (Walras, 1874/1977, Lesson 5, p.84) directs the reader's attention to the functioning the market:

> The market is a place where commodities are exchanged. Thus the phenomenon of value in exchange manifests itself in the market, and we must go to the market to study value in exchange.
>
> Value in exchange, when left to itself, arises spontaneously in the market as the result of competition. . . . The more perfectly competition functions, the more rigorous is the manner of arriving at value in exchange. The markets which are best organised from the competitive standpoint are those in which purchases and sales are made by auction . . .

The behaviour of market traders is then illustrated – buyers and sellers are shown as responding to price. An individual buyer (of commodity a) will have in mind, we are told, a demand

schedule relating all possible values of p_a and the corresponding q_a he would wish to buy. 'Now anyone at all familiar with mathematics knows there are two ways of representing this schedule mathematically.' One, as demonstrated, is by drawing a graph (ibid., pp.93-94). On the horizontal axis he places price and on the vertical, quantity demanded; the demand schedule is then represented geometrically by the curve drawn (as illustrated) through the points showing quantity demanded at different prices. Alternatively, as Walras observes, the demand schedule (for commodity a) may be represented algebraically by the equation of this curve, $d_a = f(p_a)$.

It is not until lesson 8 (ibid., p.115ff) that Walras, having discussed exchange with the aid of his downward-sloping demand curve, turns to an explanation of the shape of the curve: why, he asks, is there an increase in demand with a reduction of price? The explanation given runs in terms of what he calls 'intensive utility', i.e., the utility gained by the consumer when 'the magnitude of the sacrifice affects the quantity consumed' of the commodity in question. Side-stepping any difficulty that there might be in measuring intensive utility, Walras simply assumes that 'a direct and measurable relationship does exist' such that 'we shall find ourselves in a position to give an exact, mathematical account of the respective influences on prices of extensive utility, intensive utility and the initial stock possessed'. A diagram is provided, with quantity of commodity consumed on the vertical axis and, on the horizontal axis, quantity of intensive utility. A 'utility' or 'want' curve is drawn relating intensive utility to the amount of commodity consumed; Walras notes, 'whether the curve be continuous or discontinuous, I postulate that intensive utilities always diminish from that of the first unit or fraction of a unit consumed to that of the last unit or fraction of a unit consumed'. Then finally:

> If, now, we let the term *rareté* designate the intensity of the last want satisfied by any given quantity consumed of a commodity, then the curve . . . becomes our individual's rareté curve, considered as a function of the quantity of [the commodity] he consumes. . . . We must postulate, I repeat, that rareté increases as the quantity possessed decreases, and vice versa.

It will be noted – using more familiar terminology – that here we have a diagram showing marginal utility diminishing as quantity consumed increases, but *drawn with quantity (the independent variable) on the vertical axis and marginal utility on the horizontal.*

Diminishing marginal utility is explained, somewhat simpler terms, in Appendix I – Geometrical Theory of the Determination of Prices (Walras, 1874/1977, pp.461ff), which, in the third edition, was added to Walras's text:

> We may say in ordinary language: 'The want which we have for things, or the utility which things have for us, diminishes gradually as consumption increases. The more a man eats, the less hungry he is; the more he drinks the less thirsty, at least in general and apart from certain deplorable exceptions. The more hats and shoes a man has, the less need he has of a new hat or a new pair of shoes; the more horses he has in his stables, the less effort he will make to procure another horse . . .' But in mathematical terms we say: 'the intensity of the last want satisfied is a decreasing function of the quantity of the commodity consumed'.

Again, the associated diagrams have quantity on the vertical and rarete (marginal utility) on the horizontal axis. What, we may ask, is going on here? Why is Walras apparently ignoring convention and representing the independent variable – quantity consumed – on the vertical axis? We suggest that the answer lies in the fact that, from an *overall perspective* (as hinted at in Walras's initial observations about markets to which we referred earlier) Walras's analytical focus was on ultimately on the attainment of market equilibrium, both in the individual market, and in terms of general equilibrium, throughout the whole interdependent set of markets within the economy – *specifically on the attainment of equilibrium via quantity adjustments in response to price signals.*

Walras's stylised account of adjustment to equilibrium – the famous *tâtonnement* process – depicts a (hypothetical) phase by phase process of attainment of market-clearing equilibrium: trial prices are set ('cried') at each stage of the process; agents respond by adjusting quantities demanded and supplied to these prices presented to them. What is envisaged is a sort of auction, in which a notional 'auctioneer' calls out a set of prices for all commodities being traded, and traders make known their demands and supplies at the prices quoted. If quantities demanded and supplied are not equal, a different set of prices is proposed, and traders adjust their quantities demanded and supplied to the new relative prices. As Walras puts it (p.242): 'When a price is cried, and the effective demand and offer corresponding to this price are not equal, another price is cried for which there is another corresponding effective demand and offer . . .' Thus (pp.169-

170), in the context of a model with m commodities (and m-1 prices in terms of a numeraire), the auctioneer keeps changing prices when markets are found not to clear:

> If the total demand equalled the total offer of each and every commodity . . . the exchange would take place [at the given prices] and the problem [of achieving equilibrium] would be solved. Generally, however, the total demand will not equal the total offer of each and every commodity . . . What will happen on the market then? If the demand for any one commodity is greater than the offer, the price of that commodity in terms of the numéraire will rise; if the offer is greater than the demand, the price will fall.

And correspondingly (referring to the attainment of equilibrium relative values in a 'production' system) the agents in the market adjust quantities demanded and supplied according to the price on offer (p.476):

> We may then enunciate the following proposition which is specific to the theory of production: equality between the selling price of products and the cost of the productive services employed in their manufacture is attained by increasing the quantity of those products the selling price of which exceeds the cost of production and by d0ecreasing the quantity of those products the cost of production of which exceeds the selling price.

Our interpretation of Walras's diagrammatic convention of consistently measuring price on the abscissa and quantity on the ordinate, despite explaining diminishing marginal utility as a function of quantity, is that his graphical treatment reflected his *primary* analytical purpose, that of modelling the attainment of market-clearing equilibrium, with the process of adjustment viewed in terms of agents responding by making quantity adjustments in response to the price signals presented to them.

The other way round: quantity represented as the independent variable

H H Gossen (1810-1858) is recognised as another (independent) discoverer of the principle of marginal utility. In his treatise on *The Laws of Human Relations and the Rules of Action Derived*

Therefrom (1854) this lone thinker invented and employed a marginalist analysis to expound a utilitarian vision of how maximum individual pleasure and social happiness may be achieved.

> Man wants to enjoy life and makes it his goal to increase pleasures enjoyed throughout life to the highest possible level. . . . Not only is this maximisation viewed by all men without exception as life's ultimate purpose, it also is undoubtedly the real purpose of man's life, willed by his Creator. . . . [Who] has established order among His human beings. As He has forever and immutably predetermined the paths of the planets by the laws of gravitation, He has predetermined for all eternity and invariably for all men the pattern of their social existence by the laws governing their power of enjoyment.
>
> (Gossen, 1854/1984, p.3)

It was the duty, therefore, of the thinker and scholar, to reveal these laws and ensure their comprehension 'in a manner that is best for the welfare of all mankind'[6]. With respect to the particular responsibilities of the economic theorist, Gossen argued that 'national economics' (as the subject was then called in Germany) must be replaced by a 'science of pleasure' – developing the understanding necessary to guide individual behaviour and social policy. Gossen's study is accordingly framed as an investigation of 'the general laws of pleasure and economic value'.

In Gossen's treatise consumption and production are envisaged in terms of what Jevons was later to call 'a calculus of pleasure and pain'. The law of diminishing marginal utility, considered one of the fundamental laws of creation, is stated as being, in effect, self-evident.
(Gossen, 1854/1983, p.6)

> For its genuineness or truth, this revelation needs no human testimony: it confirms itself in such an indubitable manner that any proof seems superfluous [7].

[6] As summed-up by N. Georgescu –Roegen (Gossen, 1854/1983, p.lxv) what, on that understanding, is required of humanity is very straightforward: 'All that man has to do to obtain the highest bliss is to study and follow the laws established by the Creator'.

[7] Gossen nevertheless adds that 'daily life offers thousands of instances confirming both of these attributes', and supplies examples.

> In accordance, the, with this principle, an attempt must first be made to explore the laws according to which the force of enjoyment operates. Upon closer examination of the process of enjoyment, one finds the following common characteristics of all acts of enjoyment:
> A.1 The magnitude (intensity) of pleasure decreases continuously if we continue to satisfy one and the same enjoyment without interruption until satiety is ultimately reached.
> A.2 A similar decrease of magnitude (intensity) takes place if we repeat a previously experienced pleasure.

He observes that as 'the incalculable importance of this law makes it desirable to obtain the clearest possible notion of it', 'a geometric diagram may be of help here'. We immediately arrive at the first of Gossen's diagrams (1854/1984, p.9).

> Such a diagram can be drawn in the following manner: Let the time during which a pleasure lasts be represented by [the horizontal] line ab . . . [Gossen is using time (i.e. period of consumption) as a proxy for the quantity of the good consumed]; One may then imagine a perpendicular line erected at each point on the line ab, as shown . . . Let the height of each of these perpendicular lines be proportionate to the pleasure experienced . . . [then connect the end points of the verticals [from c to r]. . . . The line cr, whatever may be its particular shape, if followed from c to r, must move continuously and without interruption closer and closer to the line ab. For it is precisely this convergence, that is, the shortening of the perpendiculars on ab, that expresses geometrically the decrease of pleasure through continued enjoyment.

We may say that, given the nature of the basic principle – that each increment of pleasure diminishes as the quantity increases, there is no way in which, as a mathematician, Gossen could have presented, as an aid to understanding, his 'geometric diagram' *other than with quantity consumed measured along the x-axis and pleasure derived along the y-axis*.

Subsequently, Gossen having moved on to discussion of 'the exchange economy', with money featured as the medium of exchange, the above utility function turns directly into a demand curve. 'As everything is exchangeable for money, money becomes the common yardstick for the determination of the various p in our notation [p denoting 'quantities of pleasure'].' (Gossen, 1954/1984, p.108)

Finally, we may note that - complementing his account of how a commodity demand curve reflects the phenomenon of diminishing marginal utility – Gossen (1854/1984, pp.40-48) derives, and illustrates with the appropriate diagram, a supply curve which, on account of increasing disutility ('discomfort') of labour, he understands to be upward rising, reflecting the higher inducement required to elicit higher labour supply. ('Discomfort becomes greater and greater with uninterrupted movement' – i.e. the greater the amount of continuous effort, the more the pain.) The diagram naturally shows the dependent variable (discomfort) on the vertical axis as a function of hours worked.

It is evident that, given Gossen's identification of marginal utility (and disutility) as dependent variables, functions of quantity, his chosen diagrammatic format - in illustrating what he regarded as a basic law of creation - directly reflected his interpretation of the causal relationship and, at the same time, accorded, as he would be well aware, with mathematical convention.

Two other nineteenth century German economists, Karl Heinrich Rau (1792-1870) and Hans von Mangoldt (1824-1868) may be mentioned.

Rau, quite independently of Cournot, introduced a supply and demand diagram in a Note and developed a more elaborate version in a well-regarded text book (Rau, 1841)[8] which went into several editions. He placed quantity on the horizontal axis and price on the vertical. He used this construction to demonstrate the stability of market-clearing equilibrium; his argument was to the effect that inequality of quantities demanded and supplied would, as the case might be, cause price to rise or fall until equilibrium was restored.

Hans von Mangoldt (a more important theorist), adopted Rau's diagram, twenty years after its original publication, as the basis of his own demand and supply analysis (von Mangoldt, 1863)[9]. His stability analysis, like that of Rau, focused on the price-equilibrating role of excess demand or supply (see Humphrey, 1992, p.10). Mangoldt interpreted the demand curve as reflecting the principle of diminishing marginal utility, noting that the height of each point on the curve

[8] See Humphrey, 1992, pp.5-6 regarding Rau.
[9] See ibid., pp.10-151, regarding Mangoldt.

represents the marginal utility of the corresponding quantity. As regards supply curves he was (see again Humphrey, 1992, p.11) 'the first to draw such curves with different shapes depending on the behaviour of costs of production'; his analysis (supply price as a function of volume of output) comprehended all sorts of cases – of constant, increasing and decreasing costs, taking account both of economies and diseconomies of scale.

Again we can say that, in the cases these two German economists, the chosen diagrammatic representation of demand and supply - quantity on the horizontal axis - accords with mathematical convention by matching their analytical identification of demand price, supply price and market price as *dependent* variables, the values of which were the objects of explanation[10].

Jevons, W. S. (1835-1882), considered to be one of the principal founders of neoclassical economics, was yet another, independent discover of the concept of diminishing marginal utility. This 'law' he first proposed in a paper presented to the British Association in 1862; he further developed the marginalist approach in his *Theory of Political Economy* (1871). Although, when he published these expositions of diminishing marginal utility and its implications, he was unaware of the contributions of Gossen or of Dupuit, he developed his analysis very much in the manner of Gossen, expounding a utilitarian theory of pleasure and pain.

> Pleasure and pain are undoubtedly the ultimate objects of the calculus of economics. To satisfy our wants to the utmost with the least effort – to procure the greatest amount of what is desirable at the expense of the least that is undesirable – in other words, *to maximise pleasure*, is the problem of economics (Jevons, 1871/1970, p.101).

Like Gossen, Jevons took an optimistic view of the possibility of applying mathematical methods to economics. As he famously stated in the Preface to the first edition of his *Political Economy*:

[10] Humphrey (1992, p.14) alludes to the unhappy fate of Mangoldt's diagrams: 'Mangoldt's diagrammatic analysis should have become common property to all economists by the 1870s. That it did not is attributable to one Friedrich Kleinwächter who, upon publishing a reprint of Mangoldt's book in 1871 shortly after his death, deleted the diagrams on the grounds that 'it is utterly inconceivable to me that graphs or mathematical formulae could facilitate the understanding of economic laws'. It was not until 1894 that Mangoldt's contribution was recovered (by Edgeworth).

> In this work I have attempted to treat economy as a calculus of pleasure and pain, and have sketched out, almost irrespective of previous opinions, the form which the science, as it seems to me, must ultimately take. I have long thought that as it deals throughout with quantities, it must be a mathematical science in manner, if not in language. I have endeavoured to arrive at accurate quantitative notions concerning utility, value labour, capital, etc., and I have often been surprised to find how clearly some of the most difficult notions, especially that most puzzling of notions *value*, admit of mathematical analysis and expression (Jevons, 1871/1970, p.44)

On the supposition that pleasure and pain can be treated as quantities of two dimensions (intensity and duration) he draws attention to the fact that, in the case of each, intensity of feeling varies with the length of time over which the feeling is experienced. This is illustrated diagrammatically (Jevons, 1871/1970, p.96).

> [Time, in minutes, is measured along Ox, the horizontal axis; along the vertical (Oy) we measure intensity of pleasure.] The quantity of pleasure each minute may be represented by a rectangle whose base is supposed to correspond to the duration of a minute, and whose height is proportional to the intensity of the feeling during the minute in question. Each of these rectangles represents the feeling of one minute. The aggregate quantity of feeling generated during the time in question will then be represented by the aggregate area of the relevant rectangles. In the case illustrated the intensity of the feeling (pleasure in this instance) is supposed to be gradually declining as time (i.e. total quantity of feeling) increases. [This representation is subsequently replaced by the more general form of a smooth 'pleasure function'.]

Then, quoting Bentham, 'pleasure' becomes 'utility' (Jevons, 1871/1970, p.102):

> By utility is meant that property in any object, whereby it tends to produce benefit, advantage, pleasure, good, or happiness (all this, in the present case, comes to the same thing), or (what comes again to the same thing) to prevent the happening of mischief, pain, evil, or unhappiness to the party whose interest is considered.

At this stage, in place of the 'pleasure function' we are presented with a corresponding utility function, quantity being measured along Ox and utility vertically. Note that, Jevons, observing that 'we must express these notions in appropriate mathematical language', proposes

(1871/1970, p.109): ‘let x signify, as is usual in mathematical books, the quantity which varies independently – in this case the quantity of commodity.’ Then, turning to the diagram, he explains:

> The law of the variation of the degree of utility may thus be represented by a continuous curve and the perpendicular height of each point of the curve above the [horizontal] line Ox represents the degree of utility of the commodity when a certain amount has been consumed. . . . The degree of utility is, in mathematical language, the differential coefficient of u considered as a function of x, and will itself be another function of x.

And adds,

> We shall seldom need to consider the degree of utility except as regards the last increment which has been consumed, or, which comes to the same thing, the next increment which is about to be consumed. I shall therefore commonly use the expression final degree of utility, as meaning the degree of utility of the last addition, or the next possible addition, of a very small, or infinitely small, quantity to the existing stock.

It is only a short step from Jevons’s utility function to his demand curve. In a monetary economy, commodity values, including the relative values of successive increments of a commodity, are estimated in terms of money. Thus the curve representing the diminishing degree of utility received by the buyer of a commodity as successive units are purchased becomes an indicator of the diminishing monetary value attached to these units. In this guise it is, of course, the demand schedule The height of the demand curve at any point along it indicates the demand price of that quantity of the commodity. It is evident that, in spelling out in such detail the steps of the argument - from pleasure to utility, and from utility to demand price – Jevons (like Gossen) was determined to ensure that the reader fully appreciated the philosophical and psychological underpinnings (of which his diagram supplies a reminder) of the demand function being derived.

Lastly, regarding Jevons’s analysis of demand and supply, we observe that he develops a (sort of) supply schedule on the basis, so to say, of inverting his theory of consumer demand, to arrive

at a theory of labour supply, in which the supply price of labour increases, because of the increasing 'pain' (disutility) experienced as hours devoted to labour increase. As graphed (Jevons, 1970, p.192) with hours of labour on the horizontal axis, and price of labour (wage rate) on the vertical, the curve, which at first dips – reflecting some initial enjoyment of work – eventually rises reflecting the need for a progressively greater inducement to accept a longer working time as hours of work increase.

In concluding our discussion of Jevons, we may note that the basic demand and supply diagram (see, for instance, Fig.8, Jevons, 1871/1970, p.192), *with quantity on the horizontal axis and price on the vertical*, very neatly depicts (as, of course, it would be expected to do) the essence of Jevons's theory of value - virtually in pictorial terms. The demand and supply curves show the fundamental functional relationships, with marginal utility and disutility depending on quantities demanded and supplied; equilibrium is shown at the point of intersection of the two curves, where quantities demanded and supplied are adjusted to the level of output at which the 'pleasure' derived from the marginal unit consumed is just matched by the 'pain' of the marginal labour input into production.

Alfred Marshall (1842-1924)

We have a particular interest in understanding why Marshall drew his demand and supply curves as he did, with price on the vertical axis - and in understanding why he did not, as might have been expected, follow the procedure of Cournot, who alone of the few predecessors whose influence Marshall acknowledged, had made use of demand and supply diagrams – diagrams which, as we have seen, set price on the horizontal and quantity on the vertical axis.

But Marshall chose not to follow Cournot's example. It may be, though, that Marshall's early study of J. S. Mill's *Principles*, through which (as Marshall himself tells us) he made his first acquaintance with economics, had some influence - in the other direction - as regards diagrammatic format. If, as a mathematician engaged in what Schumpeter (1954, p.838) calls 'corrective reformulation' of Mill's 'loose statements', Marshall happened to sketch out graphical illustrations of Mill's points about prices and markets, he would very likely - given

Mill's general mode of (in effect) treating quantity as the independent variable - have found himself placing quantity on the horizontal axis and price on the vertical. For instance: in discussing the attainment of market equilibrium, Mill (1868, Bk.III, Ch.II, p.272) states that,

> if demand and supply are unequal at any moment, competition equalises them, and the manner in which this is done is by an adjustment of the value. If demand increases, the value rises; and falls, if the supply is increased. The rise or fall continues until the demand and supply are again equal to one another: and the value which a commodity will bring in any market, is no other than the value, which, in that market, gives a demand just sufficient to carry off the existing or expected supply. [A proposition equivalent to Marshall's own later definition of 'demand price'.]

As to a possible influence of Mill on Marshall, we cannot say more than that, in formalising Mill's economics, Marshall might well have become accustomed to thinking in terms of quantity as the independent factor: but that is no more than surmise. We suspect, however, that something more positive than any such mere habit may have induced Marshall, in his own mature analysis, to introduce the concepts of 'demand price' and 'supply price', with values interpreted as functions of quantities demanded and supplied.

It is the case though that, from the very beginning, in his own work, Marshall thought in terms of price being a function of quantity rather than *vice versa*. His earliest surviving diagrams are of supply curves – illustrating various, sometimes complex, hypothetical patterns illustrating how unit production costs may vary with volume of production (see Whitaker, 1975). Supply price per unit of output, as required to cover the cost of necessary inputs, is shown to depend upon level of production. Essentially the same story is carried through to the *Principles*. A supply curve, showing cost per unit of output (the summation of several elements - materials, wages, wear and tear of machinery and buildings, interest and profit), over a range of output levels, is constructed (Marshall, 1956, p.286n). Quantity is shown on the horizontal axis and price on the vertical. Thus:

> measuring amounts of the commodity along Ox [the horizontal axis] and prices parallel to Oy', we get for each point M along Ox a line MP drawn at right angles to it measuring the supply price for the amount OM, the extremity of which, P, may be called a supply point; this price MP being made

> up of the supply prices of the several factors of production for the amount OM. The locus of P may be called the supply curve.

Marshall presents therefore a very straightforward supply curve, Ps = f(Qs), directly linking supply price to costs of production and the way in which these vary with output.

Finally, we must observe Marshall's derivation of his demand curve. If, as Backhouse (2002, p.179) tells us '(Marshall), after reading Jevons's *The Theory of Political Economy*, grafted utility theory on to his theory of supply and demand by using it to explain the demand curve' the similarity with Jevons is not to be wondered at. Like Gossen and Jevons, Marshall (1920/1956, pp.78ff) explicitly derives the demand curve from 'the law of satiable wants' or of 'diminishing marginal utility' – that initial and fundamental proposition being, as he states it, 'the total utility of a thing to anyone (that is, the total pleasure or benefit it yields him) increases with every increase in his stock of it, but not as fast as his stock increases. In other words, the additional benefit which a person derives from a given increase of his stock of a thing, diminishes with every increase in the stock he already has'. Proceeding immediately from the concept of a utility function to that of the demand function, Marshall directly derives his 'law of demand': 'the larger the amount of a thing that a person has the less, other things being equal, . . . will be the price which he will pay for a little more of it; or, in other words, his marginal demand price for it diminishes.

At this point a footnote illustration (1920/1956, pp.81-82) demonstrates the construction of the (individual consumer's) demand curve - by the same procedure as followed by Gossen and Jevons.

> Let Ox and Oy be drawn with Ox horizontally, and Oy vertically. Let pounds of tea be measured along Ox and the price of tea along Oy. Take Om, Om', Om" as quantities of tea, and draw, from Ox, vertical lines mp, m'p', m"p" . . . with p, p', p" indicating demand prices for these quantities of tea. Then p, p', p", are points on [the consumer's] demand curve for tea. [Original text abbreviated]

The downward-sloping individual demand curve thus provides a graphical depiction of the proposition that the value the consumer places on the marginal unit is a function of quantity

consumed: at any point on the demand curve the price, shown by the height of the curve, measures for the consumer in question marginal utility as determined by the corresponding quantity of the commodity. The market demand curve is then arrived at by horizontal addition of the individual demand curves.

No Marshallian anomalies

Having reviewed the diagrammatic usages of Marshall and other early users of demand and supply 'geometry' there are three things which we can say with respect to Marshall's practice of placing quantity on the horizontal and price on the vertical axis.

(i) Firstly, it is quite clear that Marshall followed convention in measuring the independent variable along the x-axis and the dependent variable along the y-axis. His analysis views quantities demanded and supplied as the determinants of the marginal value of goods purchased and of the marginal cost of goods produced. Accordingly, it is in keeping with convention that he measures quantity on the horizontal axis and price on the vertical axis. If an impression exists, because Cournot and Walras put price on the horizontal axis (and because it is perhaps common to think of quantities as functions of price), that Marshall, in drawing his diagrams, departed from mathematical convention, any such belief is certainly mistaken: it is because Marshall viewed the relationships involved from a different perspective than these authors, not because his graphics were in any way unconventional, that Marshall's demand and supply diagrams differ from those of writers such as Cournot and Walras.

(2) Secondly, even though Marshall's treatment is different from that of some other eminent authorities, it does not follow that Marshall was, so to say, the 'odd man out'. While, as compared with Marshall's procedure, Cournot, Dupuit and Walras drew their diagrams 'the other way round', the treatment favoured by Marshall was the same as that which had been adopted by Gossen and Jevons. There is of course no right or wrong way in these matters, (the relationship between the numbers is the same regardless of which variable is on what axis) nor, would it appear, was one procedure more generally favoured over the other by the pioneering theorists of neoclassical microeconomics: the simple fact is that different authors approached these issues

from different perspectives and consequently adopted the particular diagrammatic format *which accorded with their point of view.*

(3) Thirdly, Marshall had very positive reason for employing the particular diagrammatic format that he did. Marshall, and this is true also of Gossen and Jevons, saw himself as having uncovered a principle fundamental to the working of the economic system, the proper understanding of which would underpin a new vision of the functioning of the economy and of the nature of economics as a discipline. This of course was the marginal principle – the notion which informs 'the science of small increments', i.e. the analytical approach from which the basic concepts of diminishing marginal utility and increasing marginal cost derive.

All these authors had a clear vision of the importance of their intellectual project. Gossen indeed went so far as to compare himself (no false modesty there!) with Copernicus (1854/1983. Author's Preface, p.cxlvi):

> I believe that I have accomplished for the explanation of the relations among humans what a Copernicus was able to accomplish for the explanation of the relations of heavenly bodies. I believe that I have succeeded in discovering the force, and in its general form also the law of the effects of this force, that makes possible the coexistence of the human race and that governs inexorably the progress of mankind.

Jevons likewise believed that he had found the key to a new understanding and the reconstruction of economics on new, and surer, foundations. From the Preface to *The Theory of Political Economy* (1871/1970, p.44):

> In this work I have attempted to treat economy as a calculus of pleasure and pain, and have sketched out, almost irrespective of all previous opinions, the form which the science, as it seems to me, must ultimately take.

And he adds, mindful of the need to make the new ideas as clear as possible:

> Mathematical readers may perhaps think that I have explained some elementary notions, that of the degree of utility for instance, with unnecessary prolixity. But it is to the neglect of economists to obtain clear and accurate notions of quantity and degree of utility that I venture to attribute the present difficulties and imperfections of the science, and have purposely dwelt on the point at full length.

Marshall (1920/1956, p.xiv) says very much the same thing – that proper understanding of economic phenomena depends on adequate grasp of the marginal analysis:

> one of those fundamental difficulties which have underlain and troubled the economic analysis of past times' . . . [and that] 'The new analysis is endeavouring gradually and tentatively to bring over into economics, as far as the widely different nature of the material will allow, those methods of science of small increments (commonly called the differential calculus) to which man owes directly or indirectly the greater part of the control that he has obtained in recent times over physical nature.

It was a prime concern of these three authors to get readers to understand the new approach by thinking (in general) in terms of the concept of the margin and marginal substitution and (in particular) in terms of the law of diminishing marginal utility. The basic proposition is that marginal value (or cost) depends on quantity – in other words, marginal utility or marginal cost (and so demand price and supply price) are functions of quantity demanded or supplied. Presentation should be such as to give readers the 'clearest possible notion' of the key idea of the dependence of marginal value on quantity. All three authors believed diagrammatic presentation to be a valuable aid to understanding: *it would in the circumstances seem inconceivable that they should – all being familiar with mathematical convention* - draw their diagrams other than with quantity on the horizontal axis and price on the vertical.

And it was worthwhile conforming to convention by measuring the independent variable along the x-axis and the dependent along the y-axis. That format facilitates interpretation of the diagram through awareness of the author's perspective: the attention of the reader is directed to the author's understanding of the key functional relationships. Note that with the Marshallian usage, reading along the demand curve *directly* illustrates the principle of diminishing marginal utility, the foundation stone of the whole marginalist conception - increased quantity implies

lower marginal value and demand price. By contrast, with price treated as the independent variable, the link from price to quantity is less intuitively obvious - a price change achieving a quantity change only *indirectly* as quantity is adjusted to keep the value of the marginal unit in line with the altered price [11].

The other side of the coin here is that the authors who did not, in diagrammatic terms, treat quantity as the independent variable and instead put price on the horizontal axis, would appear to have had different intellectual priorities. Cournot, as we have seen, was not interested in offering any explanation of the observed relationship between price and quantity demanded. He was not prepared to have any analytical truck with such things as 'quantities' of pleasure or utility, and so refused to go down the road of pleasure or utility functions. Nor does Fleeming Jenkin go into such matters. It looks as if he was content to take the principle of diminishing marginal utility as given, and, making use of the demand and supply apparatus, concentrate on specific issues such as the effect of trade union activity on wages and employment and the welfare effects of tax changes on consumers' and producers' surplus.

Dupuit's is an interesting case. Despite being credited as one of the discoverers of the principle of diminishing marginal utility, he evidently didn't think it important or interesting to explore the foundations of diminishing marginal utility via consideration of 'pleasure functions' or utility functions. While Gossen and Jevons and Marshall attached great significance to the discovery of this fundamental law and wished always to keep the quantity – marginal utility relationship to the fore, it looks as if Dupuit's attention was focused more narrowly on the practical implications of the principle as they related to the professional concerns of a civil engineer. It is perhaps not surprising that, in his reflections on demand and social value, being primarily concerned with the impact of infrastructure improvements, and associated taxes and tolls on social welfare, he thought of price, not quantity, as the independent variable, and therefore put price on the horizontal axis.

[11] For example, Humphrey (1992, p.17) observes that Jenkin drew his demand curves (price on the x-axis) 'with a negative slope indicating that lower prices are required to compensate for diminishing marginal utility of additional units bought'.

That brings us finally to 'Whitaker's allegation' that Marshall, having begun his analysis of prices and markets with consideration of the supply side, and having introduced functions showing supply price as depending on quantity of output produced, found himself – for the sake of consistency with his supply curve – reluctantly forced into representing the demand curve in corresponding fashion with quantity demanded on the horizontal axis, a presentation said to conflict with the supposed 'natural' view of demand as a function of price. Whitaker seems to imply that Marshall was not too happy with this representation of demand. Having examined Marshall's analysis of demand and his construction of his demand curve, we suspect that Whitaker may have read too much into Marshall's footnote comment on lay views regarding the relationship between quantity demanded and price. What Marshall himself said (1920, p.457n) was this:

> Strictly speaking, the amount produced and the price at which it can be sold, are functions one of another, account being taken of the length of time allowed for the evolution of appropriate plant and organization for production on a large scale. But in real life, the cost of production per unit is deduced from the amount expected to be produced, and not *vice versa*. Economists commonly follow this practice; and they follow also the practice of business life in inverting this order with regard to demand. That is, they consider the increase of sales that will follow from a given reduction of price, more frequently than the diminution of price which will be required to effect a given increase of sales.

Marshall, that is to say, recognised that businessmen (economists also) commonly think in terms of quantity demanded as a function of price. No doubt Marshall would have *liked* his chosen treatment of demand correspond, as he supposed was the case with supply, to normal business usage, but if Whitaker implies that Marshall considered the representation (as actually chosen) of quantity demanded as a function of price to be 'unnatural', we think that view too strong. In other words, although we agree that Marshall may have regretted that his representation of the demand curve could not be, as for public acceptability he might have liked it to be, we suggest that, on the other hand, he could not but have recognised the positive case for treating price as the dependent variable. Indeed, we believe that is precisely how - given his derivation of the demand function in terms of diminishing marginal utility – he would be expected, in the manner of several other pioneers of the marginal utility theory, to draw the demand curve. Representing

quantity as the independent variable, the determinant of demand price and supply price is the clearest possible way of emphasising to the reader the key role of the marginal principle. It doesn't seem at all plausible to suggest that, had Marshall's freedom of action not been constrained by the particular representation of supply curve he was employing, he might have adopted Cournot's demand curve with quantity demanded measured on the vertical axis.

Conclusion

We have reviewed how Marshall, and how a number of his contemporaries and predecessors chose to draw their demand and supply diagrams. Initially at least no common practice or usage emerged. Some authors measured price along the horizontal axis, others placed quantities demanded and supplied on that axis. Obviously, as one can read, or graph, the functional relationship depicted by a demand or supply curve either way - quantity as a function of price, or price as a function of quantity – *in a sense* it doesn't really matter how these diagrams are drawn, the relationships between the numbers are the same. But we believe nevertheless that the authors in question were not indifferent between the alternative representations – it looks very much as if all (and they were all mathematically aware) made a deliberate decision about which variable (price or quantity) should, as the independent variable, be placed on the horizontal axis. In other words, the observed differences in graphical practice amongst these pioneers of microeconomic analysis did not occur simply at random.

If following the diagrammatic convention facilitates interpretation, then these various theorists drew their diagrams as would best convey the conception they had in mind. Gossen, Jevons and Marshall wanted to demonstrate how the principle of diminishing marginal utility explained the shape of the demand (and supply) curve – thus their diagrams identified quantity changes as the operative factor determining, via diminishing marginal utility, changes in demand price. On the other hand, Dupuit, interested in the welfare effects of cost changes, resulting from improved communications or from variations in tolls or taxes, treated price as the independent variable. While Marshall's treatment did differ from that of other eminent pioneers of the marginal analysis, such as Cournot, Dupuit and Walras that is no reason for criticism: the treatment favoured by each author was appropriate to his own interest and analysis. There is no reason why

they should all have followed the same road. No sign of convention flouted. Nor does there seem to be reason to think that Marshall might have followed the presentation of Cournot or Dupuit had he not (allegedly) found himself trapped by the representation of supply to which he was already committed.

As to the ‘anomaly’ with which we started, while it may be due to sloppy contemporary exposition, no blame can be attached to any of the pioneers of ‘economic geometry’ The inventors of the demand and supply cross diagram approached the questions at issue with different priorities and from different angles, with the result that differences in diagrammatic presentation emerged. Any consequent complication was natural and unavoidable.

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