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TYPE II ERRORS IN IO MULTIPLIERS

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Abstract

This paper compares methods for calculating Input-Output (IO) Type II multipliers. These are formulations of the standard Leontief IO model which endogenise elements of household consumption. An analytical comparison of the two basic IO Type II multiplier methods with the Social Accounting Matrix (SAM) multiplier approach identifies the treatment of non-wage income generated in production as a central problem. The multiplier values for each of the IO and SAM methods are calculated using Scottish data for 2009. These results can be used to choose which Type II IO multiplier to adopt where SAM multiplier values are unavailable.

"Multiplication/ that's the name of the game/ and each generation/ they play it the same" Bobby Darin (1961).

1. Introduction

This paper compares methods for calculating Input-Output (IO) Type II multipliers. These are formulations of the standard Leontief demand-driven IO model which attempt to endogenise at least a part of household consumption. This is done essentially through a two-step process. First, a link is made between income generated in production and household income. Second, the endogenous change in household income then stimulates corresponding changes in household consumption.

In this discussion the standard IO assumptions that hold in production are assumed to be extended to the generation of household income and expenditure. These assumptions are that there are no supply constraints and that there are fixed coefficients in the linear production and consumption functions. This implies that all responses to changes in demand occur through changes in output, with no changes in prices, and that these responses are linear, with average and marginal values being identical. There are two basic IO Type II multiplier methods that are available in the literature. We label them the Miller and Blair (M+B) and Batey approaches. The Batey approach has two variants identified here as Batey1 and Batey2 (B1 and B2).

2. IO Type I Multipliers

The Type I multiplier incorporates the direct and the indirect effect associated with production for final demand. It is derived as follows:

(1) $Ax + f = x$

where there are n sectors, A is the nxn matrix of technical production coefficients, f is the nx1 vector of final demands and x is the vector of outputs.[1] Subtracting Ax from both sides of equation (1) gives:

[1] A table of all variables used in this paper is given in Appendix 1.

$$(2) \quad f = [I - A]x$$

Premultiplying both sides of (2) by $[I\text{-}A]^{-1}$ produces the familiar IO equation that links output to final demand:

$$(3) \quad [I - A]^{-1} f = x$$

In this case $[I\text{-}A]^{-1}$ is the Type I Leontief inverse where the representative element $\alpha_{i,j}$ is the direct and indirect output in sector *i* associated with a unit of exogenous final demand in sector *j*. Summing the elements of column *j* gives the Type I multiplier for sector *j*, M_j^I. This is the total output across all sectors associated with a unit increase in exogenous demand for the output of sector j. If there are n sectors it is given as:

$$(4) \quad M_j^I = \sum_{i=1}^{n} \alpha_{i,j}$$

Note that equation (3) can be interpreted as an accounting identity, in that any initial set of IO accounts can be manipulated in this way, so that the actual vector of outputs is attributed to actual final demand. Moreover, if all the relevant assumptions are imposed, then equation (3) can be used as a model in which changes in final demand drive, in a linear and deterministic manner, changes in total output.

3. IO Type II Multiplier

In the Type I model, all household consumption expenditure on domestic goods is included in exogenous final demand. The Type II multiplier seeks to endogenise some or all of household consumption. This task presents two central problems, both relating to the limited information available in the IO accounts. The first is that it is not possible to track fully all the income that is generated in production which goes, either directly or indirectly, to households. The second is that with the data given in the IO accounts, accurate household coefficients cannot be calculated.

To begin, although household income should be linked to all factor income that is generated in production, the conventional IO Type II approaches tie endogenous household consumption solely to wage income. The total wages, W, generated in production are straightforward to calculate. They are given as:

$$(5)\quad W = wx$$

In equation (5) *w* is the *1xn* vector of wage coefficients, where the ith element is the wage payment in sector i divided by the total output of that sector. In the Type II multiplier, labour demand is therefore generated in the same way as the demand for any other intermediate input.

The key aspect of the Type II multiplier is that the household consumption demand vector given in the IO accounts, c, is divided into two nx1 vectors representing endogenous, c_N^Z, and exogenous, c_X^Z, household consumption expenditures. In principle, endogenous household consumption expenditure is expenditure funded by income generated in production, whereas exogenous household expenditure is financed through savings, transfers (pensions, welfare payments etc). Each of the three multiplier methods, identified by the superscript Z, does this breakdown in a different way, but in all:

$$(6)\quad c = c_N^Z + c_X^Z$$

In the Type II IO context, the ith element of the c_N^Z vector is equal to the appropriate consumption coefficient, $\varphi_{N,i}^Z$, times what is taken to be the endogenous household income, Y_N^Z. Therefore:

$$(7)\quad c_N^Z = \varphi_N^Z Y_N^Z$$

where φ_N^Z is the nx1 vector of endogenous household consumption coefficients.

Combining equations (2), (5),(6) and (7) and presenting in matrix form gives:

$$(8)\quad B^Z j^Z + f^Z = j^Z$$

where B^Z is an (n+2)x(n+2) matrix, and where f^Z and j^Z are n+2 column vectors, given as

$$B^Z = \begin{bmatrix} A & 0 & \varphi_N^Z \\ w & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \; f^Z = \begin{bmatrix} f - c_N^Z \\ 0 \\ 0 \end{bmatrix} \text{ and } j^Z = \begin{bmatrix} x \\ W \\ Y_N^Z \end{bmatrix}$$

Using the familiar matrix inversion, the Type II accounting identity that corresponds to equation (3) in the Type I formulation:

$$(9)\quad \left[I - B^Z\right]^{-1} f^Z = j^Z$$

The matrices and vectors A, w and c do not vary across different IO Type II methods. However the φ_N^Z vector of endogenous household coefficients does and this will also imply variations across multiplier methods in the endogenous final household consumption demand vector, c_N^Z .

As with the Type I multipliers, if $\beta_{i,j}$ is the coefficient in the *i*th row and *j*th column, the multiplier value for sector *j* is the sum of the first n elements of the *j*th row. That is to say:

$$(10)\qquad M_j^Z = \sum_{i=1}^{n} \beta_{i,j}$$

Again, this is the impact on total output of a unit change in the exogenous final demand for the output of sector *j*.

3.1 Miller and Blair (1985)

Miller and Blair endogenise all household consumption. That is to say, $c_N^{M+B} = c$ and total household income, Y, consists solely of wages, so that $Y = W$. The ith element of the endogenous household consumption vector, $\varphi_{N,i}^{M+B}$, is therefore calculated as the ith element of the total domestic household consumption vector, c_i, divided by the total wage payment, W, so that:

$$(11) \qquad \varphi_N^{M+B} = \frac{c}{W}$$

The primary problem for the M+B method is that typically only around 60% of all household income comes from wages. Moreover, perhaps more critically, some elements of household consumption, such as pensions and some government transfers, are conventionally treated as being exogenous, independent of income generated in current production. This issue is fudged in the example given in Miller and Blair (1985, p. 28) where the sum of household consumption is given as arbitrarily equal to the total wage payment. We would expect the M+B method to overestimate the true Type II multiplier values.

3.2 Batey (1985)

The Type II multiplier approach outlined in Batey (1985) acknowledges the existence of exogenous household expenditure. The Batey method attempts to capture the addition to household consumption that comes through changes in wage income alone. In the first variant of the Batey method, which we label Batey1, the ith coefficient in the household consumption vector is the corresponding entry in the IO accounts divided by total household income, *Y*, so that:

$$(12) \qquad \varphi_{N,i}^{B1} = \frac{c_i}{Y}$$

There are a number of drawbacks to this procedure. The first is the obverse of the problem facing Miller and Blair. M+B can be criticised for assuming that all income to households comes from wages. However, a criticism of Batey1 is that there are sources of income generated in production, apart from wages, that enter household income either directly from other value added or indirectly through elements of corporate income that are subsequently distributed to households. Therefore endogenising household expenditure as that consumption funded directly by wage income will give a multiplier that is too low. A second problem is that the total household income is not a figure that is given in the IO accounts. It needs to come from some other source.

A variant of the Batey approach, that we label Batey2, retains the spirit of the Batey method but relies solely on data from the IO accounts. In this case, the vector of household coefficients, φ_N^{B2}, is constructed by dividing the entries in the household consumption column in the IO accounts by total household consumption, *C*. This implies that the ith element of the vector of coefficients equals:

$$(13) \qquad \varphi_{Ni}^{B2} = \frac{c_i}{C}$$

There are two main problems in this case. The first is that, as with Batey1, the method does not incorporate non-wage household income generated in current production. However, on the other hand, in calculating the consumption coefficients it ignores all the household income not spent on domestic and imported goods and services. Therefore it does not take into account expenditure by consumers on some taxes, savings and other transfers. By ignoring the non-wage elements of income generation in production the multiplier will be too small. However, in ignoring income not spent on consumption, the multiplier will be too big.

4. Social Accounting Matrix Multipliers

It is clear that there is no correct way to identify the extent to which output is generated by endogenous household expenditure using just the IO accounts, if by this we mean the consumption financed by factor incomes resulting from current production. This remains true even if the IO accounts are augmented by information on total household income, as in Batey1. The reason is straightforward. IO accounts fail to identify the way in which the flows of income earned by factors of production reach households. However, a multiplier that endogenises household consumption based around a Social Accounting Matrix (SAM) can track such income flows, if the same sort of assumptions concerning linearity and exogeneity are made as imposed in IO.

The SAM multiplier is based around a Social Accounting Matrix, a set of disaggregated economic accounts. These have the IO accounts at their core but also track the income to and expenditures from non-production accounts, such as the household, corporate, government, capital and external accounts (Round, 2003). In addition to production, the SAM multiplier typically endogenises the wage, other value added, household and corporate accounts. That is to say, government, capital and external expenditure is taken to be exogenous. This includes government transfers.

In the SAM multiplier, total other value added, Π, is determined in exactly the same way as wages in the Type II IO:

(14) $\Pi = \pi x$

where π is an nx1 vector whose ith value is the other value added in the ith sector divided by the total output of that sector. A share of value added, ρ^Y goes directly to households and a share ρ^R goes to corporations. Subsequently a share of corporate income, r^Y, is transferred to households. This means that in the SAM multiplier, corporate, R, and household income are given as:

(15) $R = \rho^R \Pi + T^R$

(16) $Y = W + \rho^Y \Pi + r^Y R + T^Y$

where T^R and T^Y are exogenous transfers to the corporate and household sector from the government and external sectors. Finally for household expenditure the appropriate coefficients are the Batey1 values. Combining equations (3),(5),(12),(14), (15) and (16) and expressing this in matrix form produces:

$$(17) \qquad S\begin{bmatrix} x \\ v \end{bmatrix} + \begin{bmatrix} f-c \\ f_V \end{bmatrix} = \begin{bmatrix} x \\ v \end{bmatrix}$$

where the S is the (*n*+4)x(*n*+4) matrix of the form:

$$\begin{vmatrix} A & 0 & 0 & \varphi_N^{B1} & 0 \\ w & 0 & 0 & 0 & 0 \\ \pi & 0 & 0 & 0 & 0 \\ 0 & 1 & \rho^Y & 0 & r^Y \\ 0 & 0 & \rho^R & 0 & 0 \end{vmatrix}$$

where f_V is the 4x1 vector of exogenous income transfers and *v* is the 4x1 vector of factor and institutional incomes, so that:

$$f_V = \begin{bmatrix} 0 \\ 0 \\ T^Y \\ T^R \end{bmatrix}, \quad v = \begin{bmatrix} W \\ \Pi \\ Y \\ R \end{bmatrix}$$

Through the standard matrix inversion:

$$(18) \qquad [I-S]^{-1}\begin{bmatrix} f-c \\ f_V \end{bmatrix} = \begin{bmatrix} x \\ v \end{bmatrix}$$

The multiplier outlined here endogenises both the household and the corporate sector. Therefore, the direct link between household income and other value added, as well as the flow of other value added through corporations to households is endogenised in the SAM multiplier. As we have stated already, traditionally, the government, capital, and external sector are treated as exogenous in the model (Round, 2003).[2]

Again if the element in the ith row and the *j*th column of the SAM inverse is represented as $\sigma_{i,j}$ then the SAM multiplier value for sector j , M_j^S , is the sum of the first n elements of row *j*, given as:

$$(19) \qquad M_j^S = \sum_{i=1}^{n} \sigma_{i,j}$$

Again, this measures the system-wide change in total output generated by a unit increase in exogenous final demand for the output of sector j.

5. Analytical Comparison of Multiplier Values

If the SAM framework is accepted as the most appropriate way to endogenise household consumption in a manner consistent with the Input-Output approach, none of the standard IO Type II multiplier methods is correct. Equations (20) and (21) adjust the B^Z and S marices shown in equations (8) and (17) so that their structures are harmonised in order to better identify the differences.

$$(20) \qquad \bar{B}^Z = \begin{bmatrix} A & 0 & 0 & \varphi_N^{B1} \\ w & 0 & 0 & 0 \\ \pi & 0 & 0 & 0 \\ 0 & \kappa^Z & 0 & 0 \end{bmatrix}$$

where $\kappa^{B1} = 1$, $\kappa^{B2} = \frac{Y}{C}$ and $\kappa^{M+B} = \frac{Y}{W}$, and

[2] There is an argument for endogenising other elements of these disaggregated accounts. In the present context, it is sometimes argued that endogenising transfers, particularly those linked to population and employment status, increases the accuracy with which household consumption is modelled (Batey, 1985; Batey and Madden, 1983; Batey and Weeks, 1989).

(21) $$\bar{S} = \begin{bmatrix} A & 0 & 0 & \varphi_N^{B1} \\ w & 0 & 0 & 0 \\ \pi & 0 & 0 & 0 \\ 0 & 1 & \rho^Y + \rho^R r^Y & 0 \end{bmatrix}$$

Each of the four rows and columns in the $\bar{B}^Z$ and $\bar{S}$ matrices represent receipts and expenditures of the industries, labour, other value added and household accounts. Note that the first three rows of these matrices are identical. They use the same *A* matrix and *w*, π and c_N^{B1} vectors of coefficients. The two matrices differ solely in the fourth row which identifies the sources of income entering the household account.

In the $\bar{B}^Z$ matrix one adjustment is the addition of the other value added account. However, its impact is trivial. Although we can identify the other value added generated in production, the destination of other value added expenditure is unknown in the IO accounts. Therefore the other value added column, column three in $\bar{B}^Z$, only has zero elements. The second change is more interesting. In equation (8) the different Type II multiplier formulations are identified by their different household consumption coefficients. However, it is straightforward to show that this can be translated to a differences in the level of wage income transferred to households, combined with the household consumption coefficients used in Batey1 and the SAM multipliers .

The consumption coefficient $\varphi_{N,i}^{B1}$ is defined in equation (12) and $\varphi_{N,i}^{B2}$ in equation (13). Using these equations, the coefficients $\varphi_{N,i}^{B2}$ can be expressed as:

(22) $$\varphi_{N,i}^{B2} = \frac{c_i}{C} = \frac{c_i}{Y}\frac{Y}{C} = \varphi_{N,i}^{B1}\kappa^{B2}$$

where $\kappa^{B2} = \frac{Y}{C}$. Applying a similar procedure to equations (11) and (12):

(23) $$\varphi_{N,i}^{M+B} = \varphi_{N,i}^{B1}\kappa^{M+B}$$

where $\kappa^{B2} = \frac{Y}{W}$.

Equations (22) and (23) show that the Miller and Blair and Batey2 household consumption coefficients are simply scalar multiples of the Batey1 coefficients, which are the coefficients also used in the SAM multipliers. The different Type II IO multipliers can therefore solely be represented by differences in the relationship between the change in wage income and the subsequent change in effective household income.

Given that, in the Scottish data, $Y > C > W$, the relative values of values of κ^Z for Scotland are $\kappa^{M+B} > \kappa^{B2} > \kappa^{B1} \equiv 1$. Note that this implies the seemingly illogical position that in the Batey2 and M+B multiplier measures, more than 100% of the wage income is assumed to be transferred to household income. However, as has been remarked already, in the B^Z matrix there is no transfer of other value added to household income. Therefore some overweighting of wage income could be justified on this basis. These observations have a number of implications. Begin with the IO Type II multipliers. For each industry, their values can be ranked in the same order as their κ^Z values. That is to say, for Scotland for any industrial sector, *i*, $M_i^{M+B} > M_i^{B2} > M_i^{B1}$. However, a comparison between the IO Type II and the SAM multiplier values is a little more complex.

The Batey1 multiplier value is always lower than the SAM multiplier: for any sector, *i*, $M_i^S > M_i^{B1}$. This is apparent from a comparison of the $\bar{B}^{B1}$ and the $\bar{S}$ matices given in equations (20) and (21). The only difference in the two matrices is the additional elements in the SAM matrix, $\bar{S}$, linking household income positively to other value added.

On the other hand, the value of the Miller and Blair Type II multiplier will generally higher than the corresponding SAM value. The sum of the M_i^{M+B} values, weighted by their associated final demands, will be greater than the corresponding weighted sum of the

SAM multipliers. This is because in the accounting identity (equation 9) the M+B multiplier endogenises all household income through directly linking all household income linearly to wage payments. But, in general, there are exogenous elements in household income, so that T^Y is positive in equation (17). This means that the M+B method typically overcompensates for not directly including the link between household income and other value added generated in production. However, this does not mean that M_i^{M+B} is necessarily greater than M_i^S for all industries. If an industry is very capital intensive and if a significant share of other value added is transferred to household income, the SAM multiplier can be higher than M+B for particular individual industries.

Clearly the Batey2 multiplier takes an intermediate position, between the Batey1 and Miller and Blair figures. Its value relative to the SAM multiplier is wholly data dependent. The Batey2 average multiplier value and the value for individual sectors could be higher or lower than the corresponding SAM values, depending on the the extent to which the impact of wages on household income under- or over-compensates for the missing income from other value added. This in itself might reflect the level of other value added income retained in the local economy.

7. Empirical Comparison of Multiplier Values

Table 1 compares the the IO Type I, Type II and SAM multiplier values across Scottish industrial sectors for 2009. The Type II IO multipliers comprise the M+B, Batey1 and Batey2 variants. The data used are the 2009 Scottish Industry by Industry (IxI) Table (Scottish Government, 2013) and the 2009 Scottish SAM (Emonts-Holley et al., 2014). The SAM is constructed around the corresponding IO accounts, so that the multiplier values are consistent. The deviations of the IO Type II multipliers for each sector from the corresponding SAM multiplier value are given in Figure 1. The horizontal axis represent the SAM multiplier value so that all the observations for each industry are measured relative to the corresponding SAM value. Therefore the closer a line is to this axis, the better it approximates the SAM multiplier value.

Table 1: IO and SAM multipliers for Scotland

	Type I	Type II			SAM
		Miller & Blair	Batey1	Batey2	
1. Agriculture	1.608	1.996	1.802	1.918	1.964
2. Forestry planting	1.615	2.111	1.863	2.011	1.972
3. Forestry harvesting	1.961	2.517	2.239	2.405	2.367
4. Fishing	1.611	1.995	1.803	1.918	1.933
5. Aquaculture	1.625	1.956	1.790	1.890	1.916
6. Coal & lignite	1.671	2.118	1.894	2.028	1.983
8. Other mining	1.435	1.985	1.709	1.874	1.786
9. Mining Support	1.501	1.858	1.679	1.786	1.847
10. Meat processing	1.917	2.410	2.163	2.311	2.250
11. Fish & fruit processing	1.695	2.229	1.962	2.122	2.044
12. Dairy products, oils & fats processing	1.923	2.478	2.200	2.366	2.300
13. Grain milling & starch	1.803	2.300	2.051	2.200	2.134
14. Bakery & farinaceous	1.426	2.088	1.756	1.955	1.840
15. Other food	1.609	2.189	1.898	2.072	1.980
16. Animal feeds	1.589	2.086	1.837	1.986	1.897
17. Spirits & wines	1.299	1.779	1.538	1.682	1.694
18. Beer & malt	1.367	1.814	1.590	1.724	1.746
19. Soft Drinks	1.493	2.057	1.774	1.944	1.872
21. Textiles	1.436	2.110	1.772	1.974	1.830
22. Wearing apparel	1.465	2.241	1.852	2.085	1.907
23. Leather goods	1.497	2.137	1.816	2.008	1.890
24. Wood and wood products	1.801	2.481	2.140	2.345	2.223
25. Paper & paper products	1.662	2.210	1.936	2.100	2.010
26. Printing and recording	1.378	2.232	1.804	2.060	1.883
27. Coke, petroleum & petrochemicals	1.204	1.312	1.258	1.290	1.321
28. Paints, varnishes and inks etc	1.421	1.972	1.696	1.861	1.756
29. Cleaning & toilet preparations	1.460	2.203	1.831	2.054	1.895
30. Other chemicals	1.251	2.099	1.674	1.928	1.765
31. Inorganic chem., dyestuffs &agrochem	1.314	1.939	1.626	1.814	1.716
32. Pharmaceuticals	1.349	2.018	1.683	1.884	1.776
33. Rubber & Plastic	1.491	2.266	1.878	2.110	1.948
34. Cement lime & plaster	1.594	2.257	1.925	2.124	1.997
35. Glass, clay & stone etc	1.473	2.207	1.839	2.059	1.915
36. Iron & Steel	1.401	2.067	1.734	1.933	1.803
37. Other metals & casting	1.449	2.032	1.740	1.915	1.831
38. Fabricated metal	1.481	2.251	1.865	2.096	1.941
39. Computers, electronics & opticals	1.416	1.980	1.697	1.866	1.767
40. Electrical equipment	1.483	2.183	1.832	2.042	1.896

41. Machinery & equipment	1.519	2.304	1.911	2.146	1.983
42. Motor Vehicles	1.515	2.178	1.846	2.045	1.907
43. Other transport equipment	1.647	2.264	1.955	2.140	2.026
44. Furniture	1.574	2.284	1.928	2.141	1.999
45. Other manufacturing	1.403	2.301	1.851	2.121	1.913
46. Repair & maintenance	1.427	2.164	1.795	2.016	1.877
47. Electricity	2.053	2.405	2.229	2.335	2.345
48. Gas etc	1.260	1.544	1.401	1.487	1.482
49. Water and sewerage	1.287	1.733	1.509	1.643	1.708
50. Waste	1.493	2.195	1.843	2.054	1.941
51. Remediation & waste management	2.780	3.343	3.061	3.230	3.214
52. Construction – buildings	1.766	2.401	2.083	2.273	2.200
53. Construction - civil engineering	1.731	2.450	2.090	2.305	2.202
54. Construction – specialised	1.530	2.288	1.908	2.136	2.020
55. Wholesale & Retail – vehicles	1.335	2.116	1.725	1.959	1.815
56. Wholesale - excl vehicles	1.521	2.253	1.886	2.106	1.990
57. Retail - excl vehicles	1.352	2.139	1.745	1.981	1.858
58. Rail transport	1.764	2.582	2.172	2.418	2.265
59. Other land transport	1.400	2.033	1.716	1.906	1.810
60. Water transport	1.657	2.138	1.897	2.042	1.980
61. Air transport	1.467	1.920	1.693	1.829	1.792
62. Support services for transport	1.541	2.195	1.867	2.063	1.994
63. Post & courier	1.278	2.351	1.813	2.135	1.893
64. Accommodation	1.352	2.065	1.708	1.922	1.814
65. Food & beverage services	1.362	2.082	1.721	1.937	1.816
66. Publishing services	1.279	2.140	1.709	1.967	1.790
67. Film video & TV etc	1.454	2.100	1.777	1.970	1.869
68. Broadcasting	1.386	2.043	1.714	1.911	1.819
69. Telecommunications	1.393	2.067	1.729	1.931	1.859
70. Computer services	1.250	2.115	1.682	1.941	1.789
71. Information services	1.185	1.987	1.585	1.826	1.719
72. Financial services	1.222	1.785	1.503	1.671	1.665
73. Insurance & pensions	1.859	2.359	2.108	2.258	2.234
74. Auxiliary financial services	1.282	2.138	1.709	1.966	1.796
75. Real estate – own	1.465	1.768	1.616	1.707	1.817
76. Imputed rent	1.151	1.220	1.186	1.206	1.387
77. Real estate - fee or contract	1.503	2.198	1.850	2.059	1.971
78. Legal activities	1.241	2.069	1.655	1.903	1.781
79. Accounting & tax services	1.202	2.118	1.659	1.934	1.786
80. Head office & consulting services	1.391	2.267	1.828	2.091	1.914
81. Architectural services etc	1.437	2.239	1.838	2.078	1.953
82. Research & development	1.423	2.534	1.977	2.311	2.057
83. Advertising & market research	1.250	2.019	1.634	1.864	1.772
84. Other professional services	1.330	2.039	1.684	1.896	1.801

85. Veterinary services	1.364	2.197	1.780	2.029	1.918
86. Rental and leasing services	1.324	1.911	1.617	1.793	1.751
87. Employment services	1.301	2.351	1.825	2.140	1.918
88. Travel & related services	1.520	1.936	1.728	1.852	1.786
89. Security & investigation	1.155	2.378	1.765	2.132	1.853
90. Building & landscape services	1.388	2.329	1.857	2.140	1.964
91. Business support services	1.285	1.985	1.634	1.844	1.769
92. Public administration & defence	1.410	2.240	1.824	2.073	1.903
93. Education	1.189	2.478	1.832	2.219	1.914
94. Health	1.362	2.290	1.825	2.103	1.902
95. Residential care	1.320	2.330	1.824	2.127	1.950
96. Social work	1.236	2.496	1.864	2.242	1.959
97. Creative services	1.474	2.398	1.935	2.212	2.005
98. Cultural services	1.356	2.382	1.868	2.176	1.948
99. Gambling	1.414	1.933	1.673	1.828	1.822
100. Sports & recreation	1.407	2.332	1.869	2.146	1.950
101. Membership organisations	1.436	2.329	1.882	2.150	1.970
102. Repairs - personal and household	1.357	2.121	1.738	1.967	1.822
103. Other personal services	1.233	1.947	1.590	1.804	1.732
104. Households as employers	1.000	2.405	1.701	2.122	1.799

Source: Author's own calculations based on data in Scottish Government (2013) and Emonts-Holley et al. (2014).

Table 1 and Figure 1 show that the ordering of the Type II IO multiplier values derived from their analytical properties investigated in Section 6 are replicated in the data. For every sector, $M_i^{M+B} > M_i^{B2} > M_i^{B1}$ and the SAM multiplier is always greater than the Batey1 multiplier $M_i^{B1} > M_i^{S}$. There are two sectors, 75 and 76 - Real Estate and the Imputed Rent - where the SAM multiplier has the highest value.[3] In all other sectors M_i^{M+B} is the highest multiplier value. The Batey2 multiplier is generally above the SAM value: in only 10 of the 104 sectors is it less. There are some very pronounced positive spikes using the M+B and the Batey2 approach, where the value is large in comparison to the SAM multiplier. The three most prominent examples are for sectors 89, 93 and 96,

[3] These are 1.817 and 1.387, as against the coresponding M+B multiplier values of 1.768 and 1.220 respectively.

which are Security & Investigation, Education, and Social Work respectively. These results are driven by the relatively high share of labour in value added in these sectors.

Table 2 gives the summary statistics for the range of multiplier values, showing the maximum, minimum and mean figures. The first point to make is that if the mean values for the Type I IO and SAM multipliers are compared, the incorporation of induced activity increases the multiplier from 1.465 to 1.910. That is to say the additional output over and above the direct increase in final demand is almost doubled by including the induced household consumption effects. Second, as the evidence from Figure 1 suggests, the mean value for the Batey1 Type II multiplier is lowest, followed by the SAM, the Batey2 and finally the Miller and Blair values. The difference between the two extreme Type II mean multiplier values is 0.346. The range of Type II multiplier values is almost 40% of the most accurate measurement of additional multiplier effect, which is the SAM value 0.910.

Table 2: IO and SAM multiplier summart statistics

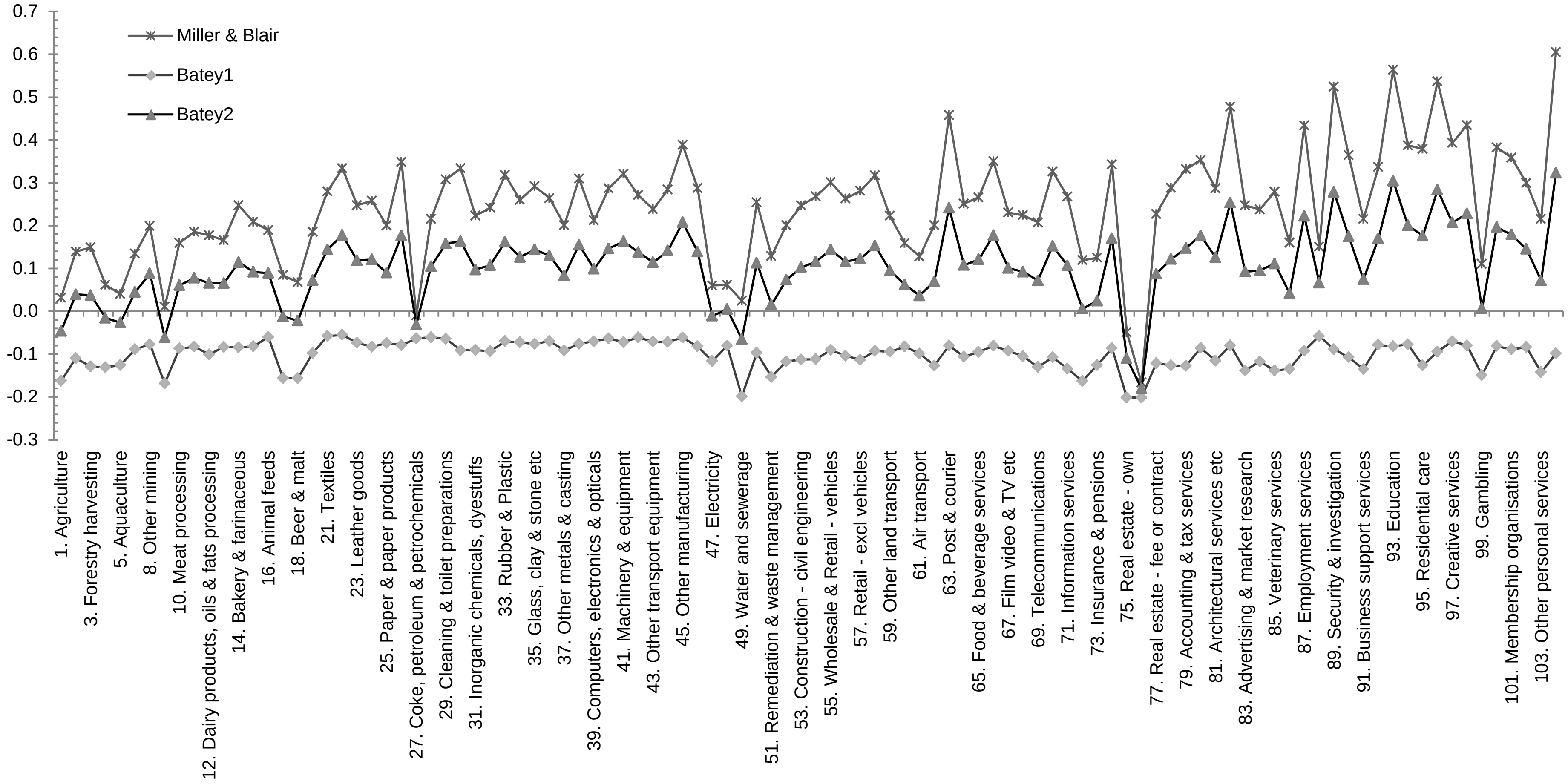
		Type II			
	Type I	Miller & Blair	Batey1	Batey2	SAM
Mean	1.465	2.156	1.810	2.017	1.910
Min	1.000	1.220	1.186	1.206	1.321
Max	2.780	3.343	3.061	3.230	3.214

Table 2 shows that the mean SAM multiplier lies within the range of the mean IO Type II values. The Batey1 figure is systematically lower than the SAM multiplier and the Batey2 and M+B approaches systematically higher. Batey1 is the Type II IO multiplier whose mean value is closest to the mean SAM multiplier, though this is only marginally closer than Batey2. The minimum and maximum multiplier values also replicate these findings. Table 3 calculates the Root Mean Square Error and Mean Absolute Error for the Type II multiplier values for individual sectors against the SAM multiplier figure. Again the Batey1 method has the lowest errors and the M+B approach the largest.

Table3: Error statistics Root Mean Square Error, Mean Absolute Error

	Miller & Blair	Batey1	Batey2
RMSE	0.201	0.077	0.099
MAE	0.131	0.054	0.062

Figure 1: Differences between the SAM and the Type II multipliers



8. Discussion and conclusion

There is complete agreement about the method used to calculate Input-Output Type I multipliers. These measure the direct and indirect output effects from a unit expansion in exogenous final demand in a particular sector. They incorporate the change in activity associated with the production of the intermediate goods that contribute directly or indirectly to the production of final demand.

Type II multipliers identify the direct and indirect effects. However, they also incorporate the impact of increased household income and subsequent consumption expenditure that accompanies any change in output. These are known as induced effects. Although this is a very common procedure, a number of different methods have been adopted in the literature. First, we believe that this variation is not widely recognised. This is potentially problematic for the interpretation of Type II multipliers, their use in modelling demand-side disturbances and the value for comparing the structural characteristics of different economies. Second, it would be valuable to standardise the Type II procedure, which requires choosing amongst the different formulations.

The first question is whether empirically this is a serious problem. The Scottish results suggest that it is. The range of Type II multiplier mean values is almost 40% of the most accurate measurement of additional multiplier effect. The second question is: which method is preferable? If the SAM multipliers embody the most complete linking of income generated in production and the subsequent distribution to households.for Scotland the mean value using the Batey1 method is closest to the mean SAM value and has the smallest mean error, even though the method systematically underestimates the SAM multiplier values. However, this method has the disadvantage that it requires information on household income that is typically not available from the IO accounts themselves.

Despite some of the models coming close to SAM multipliers, it must be acknowledged that all three Type II methods have a fundamental weakness; they all explicitly endogenise wages, and link household expenditure to these. A SAM multiplier incorporates income from other value added into household income in a way completely consistent with the standard demand-driven IO approach. It is therefore the only wholly satisfactory means of endogenising household consumption in the application of such an approach.

Appendix 1: Variable names and symbols

Symbol	Variable name
A	Matrix of technical coefficients in production
B	Matrix of Type II coefficients
C	Total household consumption
M	Multiplier value
N	Endogenous (subscript)
R	Total corporate income
S	Matrix of SAM coefficients
T	Exogenous transfers
W	Total wages
X	Exogenous (subscript)
Z	Identifier for Type II multiplier
c	Household consumption vector
f	Vector of final demands
r^K	Share of corporate income distributed to account K
v	Vector of institutional income
w	Vector of production wage coefficients
x	Vector of output
Π	Total other value added
α_{ij}	Elements of the Type I Leontief inverse
β_{ij}	Elements of the Type II Leontief inverse
φ_i	Coefficients of household consumption expenditure
σ_{ij}	Elements of the SAM inverse

κ	Income adjustment in the modified Type II matrix of coefficients
ρ^K	Share of other value added income distributed to account K
π	Vector of other value added production coefficients

References

Batey, P. (1985), “Input-output Models for Regional Demographic-Economic Analysis: Some Structural Comparisons”, *Environment and Planning A*, vol. 17, pp. 73-99.

Batey, P. and Madden, M. (1983), “Linked Population and Economic Models: Some Methodological Issues in Forecasting, Analysis and Policy Optimisation”, *Journal of Regional Science*, vol. 23, pp. 141-164.

Batey, P. and Weeks, M.J. (1989), “The Effects of Household Disaggregation in Extended Input-Output Models”, in R.E. Miller, K.R. Polenske and A.Rose, eds., *Frontiers of Input-Output Analysis: Commemorative Papers*, Oxford University Press, New York.

Emonts-Holley, T., Ross, A., & Swales, J.K. (2014), “A Social Accounting Matrix for Scotland”, *Fraser of Allander Institute Economic Commentary*, 38(1), 84-93.

Miller, R., & Blair, P. (1985), *Input-Output Analysis Foundations and Extensions*, Prince-Hall.

Round, J. (2003), *Social Accounting Matrices and SAM-based Multiplier Analysis*. Chapter 14 in F. Bourguignon and L.A. Pereira da Silva eds., *Techniques and Tools for Evaluating the Poverty Impact of Economic Policies*, World Bank and Oxford University Press.

Scottish Government (2013), *2009 IO Tables*. Retrieved 27.06.2013, from http://www/scotland.gov.uk/Resource/0042/004422350.xlsx